ENGINEERING COMPLEX SYSTEMS WITH AN EMPHASIS ON ROBUSTNESS: UTILITY-BASED ANALYSIS WITH FOCUS ON ROBUSTNESS

A Thesis

by

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ABSTRACT

Engineered system complexity continues to increase rapidly, concurrent with the requirement for the engineered system to be robust. Robustness is often considered a critical attribute of complex engineered systems, but an exact definition of robustness is not agreed upon within the systems engineering community. Lack of a clear definition, makes it difficult to develop or utilize a quantitative measure of robustness. Having a formal measure for robustness may not be considered necessary, but a lack of a specific measure results in the inability to communicate the desired level of robustness, inability to measure how various options impact robustness, and makes it difficult to measure tradeoffs between robustness and other engineering parameters.

The objective of this research is to examine robustness and how it can be attained in systems engineering. In order to accomplish this objective, data from several scientific communities is examined to develop the meaning of robustness. While definitions between and even within each community differ, a key attribute is present in each definition: A robust system needs to maintain its core functions in the presence of internal and external changes. The key component of the characteristic is that each function within a system has its own measure of robustness.

When robustness and engineering are discussed, Robust Design must be examined. The scientific community uses variance as its measure for robustness. The Robust Design method has the adverse characteristic of forcing preferences upon the designer. Examining the mean-variance approach with utility theory shows that it

imposes an increasingly risk averse position upon the designer. This position may not be compatible with the designer's true risk attitude, causing issues when applying the method.

To contend with this issue, a novel utility-based approach is suggested. The approach focuses on generating functional models of the proposed systems, which provide the designer with insight into which perturbations are relevant to the system and subsystems. Additionally this approach incorporates utility theory to allow the designer to convey their preferences. The utility-based approach allows the designer to convey their own preferences, while incorporating steps to ensure the final design is robust.

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1. INTRODUCTION

Modern systems continue to evolve and become more and more complex creating difficulty in ensuring the systems are robust. Most engineers would describe robustness as a desired quality of complex systems but the ambiguity in its definition makes it difficult to know when robustness is achieved in an engineered system. Some examples of systems that need to be robust include spacecraft and aircraft. A recent example of a complex system that failed to be robust is the Boeing 787. The new aircraft cost over $32 billion to design and is currently grounded due to battery issues [1]. The Boeing 787 experienced issues with their onboard lithium ion batteries. One plane experienced smoke from the batteries and was forced to land while another plane's batteries caught fire on the ground. The exact cause of the thermal runaway in the batteries is not known but show that the airplane is currently not robust because it is unable to perform its function under anticipated conditions. Another example of a complex system not being robust is the F-22 Raptor and its oxygen-supply system. Pilots of the F-22 were experiencing hypoxia-like symptoms during high altitude flights causing several close calls [2]. It was discovered that a pressure valve on the flight vest and carbon filter caused the issues with the oxygen-supply system [3]. The issues were corrected by changing the valve, filter, and adding a backup oxygen system. The initial design of the F-22 resulted in a system that was not robust to its desired operating conditions (high altitude and high g). Both the Boeing 787 and F-22 show that robustness in systems engineering needs to be examined in further detail.

The lack of a concrete definition makes it difficult to develop a quantitative measure to assess the level of robustness a system achieves and difficult to produce design methods that lead to robust systems. The absence of a concrete definition and no formal measure of robustness leads to the inability to perform several basic system engineering operations. These operations include, (1) communicating desired levels of robustness to others, (2) assessing the impact of various choices on robustness, and (3) trading robustness against other engineering considerations. The ambiguity in its definition and lack of agreed upon method leaves an opportunity for answering several research questions: What is the meaning of robustness in systems engineering? Is a quantitative measure of robustness needed in systems engineering? Can Robust Design be extended to engineer robust systems?

To develop a concrete definition for robustness in engineering a literature review is performed on the term. A sound background is established on robustness by examining how the term is used within several scientific communities. Two scientific communities that often deal with robustness are biology and engineering. Biological systems are considered to be highly robust and the characteristics that make them have been examined in great depth within the community. For example a human metabolic system is robust to both infections and an unstable food supply [4]. The definitions used within the biological community is consistent and has some parallels with the definitions used within the engineering community. An engineering community that must be examined when discussing system robustness and design is Robust Design.

Robust Design is a method used to develop systems that maintain performance by minimizing its variance. Robust Design or Quality Engineering initially gained popularity in manufacturing domain and has since gained popularity in other areas of engineering [5-7]. The method is sometimes used as a way to design robustness into a system. A key issue with the use of Robust Design is that it imposes preferences upon designers. A key attribute of a good design method is that it does not impose preferences upon the designer [8]. These imposed preferences are examined using utility theory, showing that they may not be reasonable.

In order to overcome the shortcomings of Robust Design a utility-based approach for higher system robustness is developed. Utility theory allows designers to formulate their preferences within a utility function and allows for the comparison of designs with multiple attributes under uncertainty [9]. The elicited utility function includes the preferences of the designer, in this case if the designer is concerned about robustness it will be present in their preferences and the utility function they elicit. The approach consists of eleven steps for increased system robustness. A key aspect of the approach include performing functional modeling on the system and subsystem functions. The functional modeling provides insight into which perturbations are relevant and need to be included within the system model. The goal of the utility-based approach is to provide the designer with valuable information on where additional resources such as time and money should be spent in order to develop a more robust system.

A case study is performed using the utility-based design approach. The case study examines the entry, descent, and landing of a Mars rover similar to the Mars

Curiosity landing sequence. The case study exhibits how the utility-based design approach aids in providing designers with information on relevant perturbations and where additional resources can be spent to increase robustness. Functional modeling forces the designers to take time and model what energy, material, and information interact with each subsystem. The improved design is able to maintain its function in the presence of internal and external perturbations.

Section 2 is broken down into a literature review of robustness and functional modeling. Section 3 examines variance as an inverse measure of robustness and background into utility theory. Section 4 examines the foundations of Robust Design and the preferences imposed onto the designer when this method is used. Section 5 presents the utility-based design approach for higher system robustness. Lastly, section 6 exhibits the utility-based design approach on a case study.

2. LITERATURE REVIEW

Robustness is often considered a crucial aspect of well-engineered systems but its definitions ambiguity causes issues on how to achieve it. What is the meaning of robustness in systems engineering? The purpose of this section is to determine the definition of robustness in systems engineering. The definition of robustness is analyzed by examining different scientific fields' definition of the term. The scientific areas examined include engineering, economics, and biology. Specific areas within engineering are examined in further detail allowing for an increased understanding of the term. In order to ensure adequate understanding of the scope of the term, related terms such as resilience and reliability are investigated. A general engineering definition is proposed within the section. Robustness focuses heavily on maintaining function under changes in anticipated internal and external properties.

A review of functional modeling is included in this section. Different functional models approaches are reviewed in order to understand the utility-based approach used in Section 5. The functional models used include black box models, EMS (Energy, Mass, and Signal) Function Structures, and hierarchical functional models.

2.1. **Survey of Terminology**

2.1.1. **General Robustness**

The term *robustness* is used differently in biological, engineering, economics and other scientific communities. However, the precise definition of the term often is not agreed upon within a given community. Typically, the root form of the term—the word *robust*—invokes some notion of vigor, strength, consistency, or sturdiness. Inconsistencies emerge when trying to make the term sufficiently precise to permit measuring robustness.

The existing lack of consistency among precise definitions very well may stem from the nature of the term itself. Table 1 contains the definition of robust from the Merriam-Webster dictionary [10].

Table 1: Definition of "robust" found in Merriam-Webster Dictionary.

A)	Having or exhibiting strength or vigorous health
B)	Having or showing vigor, strength, or firmness
C)	Strong formed or constructed
D)	Capable of performing without failure under a wide range of conditions

The four different definitions show how widely people apply the term robustness as an adjective. One can use it to show health, constructed strength, and ability to perform in different conditions. The common threads between each definition are that they are all positive and describe a system or thing that is strong or capable of performing.

The use of robustness can be further broken into specific scientific disciplines. Table 2 contains a representative sample of definitions for robustness from different scientific communities. Alternative definitions exist in each community, but these were found to be among the most common in their disciplines.

Table 2: Sample of definitions for robustness in different scientific communities.

Biology	Robustness is a property that allows a system to maintain its functions against internal and external perturbations [11].
Economics	The relationship between economic growth and a particular variable is 'robust' if the coefficient estimate remains statistically significant and if the theoretically expected sign under permutations of the set of conditioning variables [12].
Engineering	Robust systems should not produce radical departures from expected behavior in response to minor changes to operating input, internal state, or external environment [13].
Manufacturing	A robust system has minimal sensitivity to variations in uncontrollable influences [14].

The biological, engineering, and manufacturing definitions are similar in the respect that the system must maintain its nominal function or behavior in the presence of internal and external changes. However, only the manufacturing definition has sufficient specificity to indicate how to measure robustness (via sensitivity to variations). The economic relationship differs from the others in that it represents the relationship between economic growth and a variable. Yet it still relates to a consistency or lack of sensitivity in a relationship of interest. In order to gain more insight an in depth examination of robustness is done in both the biological and engineering fields.

2.1.2. **Biological Robustness**

Biological systems are believed to be highly robust to perturbations. This property is considered to be developed from evolution. However, even though biological systems are robust to most perturbations, they are still vulnerable to certain failures. An example of this is the human metabolic system, which is robust to infections and unstable food supply but susceptible to low-energy utilization lifestyles [4]. Biological systems contain several factors that contribute to their robust performance. These characteristics include redundancy, feedback control, and modularity [15]. Redundancy allows a system to perform a specific function through alternative ways. In biological systems redundancy is achieved by having several systems capable of performing the same function. Feedback control allows the system to regulate an output by comparing it to a reference.

Modularity implies that cells are "semiautonomous entities" [16]. Cells with modularity exhibit a high number of internal function connections and a low number of external function connections. This autonomous capability allows cells to be less influenced by the environment due to their low number of external environment connections. This lower susceptibility to changes in the environment in turn increases the cell's and system's robustness to environmental perturbations.

Some view robustness as involving a tradeoff between fragility, performance, and resources [11]. The tradeoff between robustness and fragility is illustrated in a simple example shown in Figure 1. The fire prevention plan shown in A is adequate against forest fires moving from west to east but susceptible to forest fires moving from south to north. Plan B places the vegetation in a circular fashion around the city protecting the city from fire in all directions. The drawback to this plan is that if the vegetation does catch fire the entire city will be surrounded by fire. Plan C calls for the removal of all vegetation around the city, allowing the city to be completely protected from forest fires but susceptible to other natural disasters such as flooding. This example shows that improving the systems robustness against a certain perturbation can cause susceptibility in another perturbation. Other methods for defending the city from wildfire can be developed but these are used as a simple example.

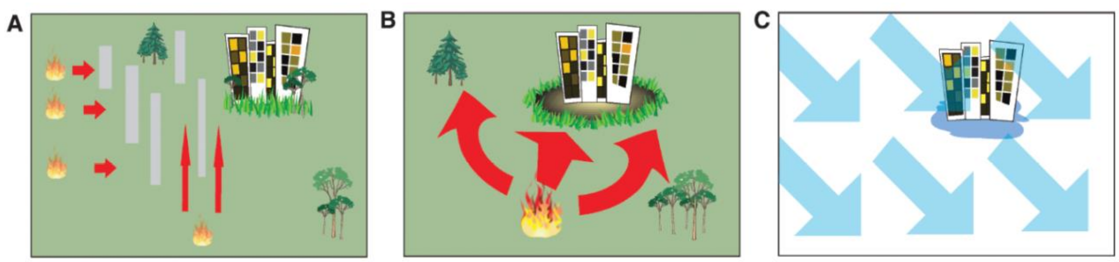

Figure 1: City fire countermeasure plan showing the tradeoff between robustness and fragility [11].

Kitano proposes a metric for robustness that gives a different robustness score for different combinations of the system feature being perturbed and the degree of perturbation [11]. This allows one to compare two or more systems as well as to understand whether certain features or functions are more or less robust to perturbation. Figure 2 is an example of how Kitano proposes to visualize the results to compare systems. In this case, Systems A and B have the same number of failures (matrix elements colored red), but System A has better overall performance retention than System B (more matrix elements are colored dark blue).

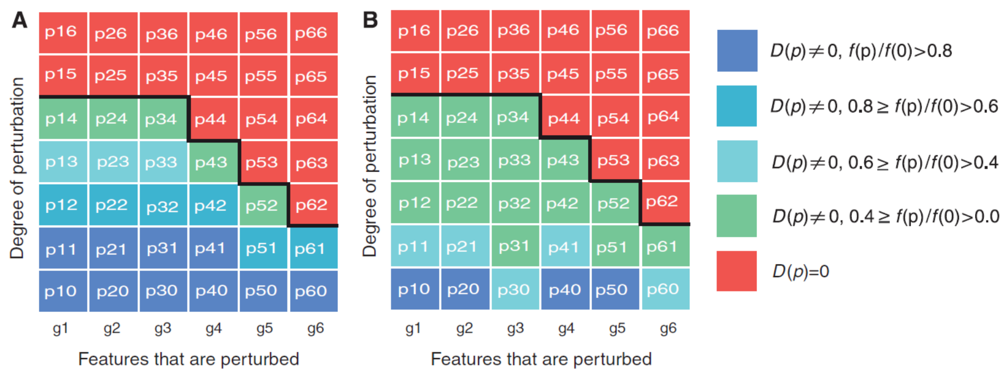

Figure 2: Degree of perturbation versus features that are perturbed [11].

In this notation, $D(p)$ is an evaluation function for how well a particular system function performs under perturbation. For a system, S, and a function, a, under a perturbation p, its definition is shown in Equation (1):

$$D_a^s(p) = \begin{cases} 0 & \text{if system failure} \\ \dfrac{f_a(p)}{f_a(0)} & \text{if no failure} \end{cases} \tag{1}$$

where a system failure means the intended functionality is not met , $f_a(\cdot)$ is a performance metric for system function a, $f_a(0)$ is the performance under nominal conditions, and $f_a(p)$ is the performance under perturbation p.

From this basis, Kitano defines the robustness of system function a as Equation (2):

$$R_a^s = \int_P \psi(p) D_a^s(p) dp \tag{2}$$

where P is the set of possible perturbations and $\psi(p)$ is the probability of perturbation p. Thus, Kitano's robustness score is the expected value of evaluation function $D(p)$.

Each system function will have a different evaluation function value and therefore a different robustness value. An advantage of this approach is it allows of each function to be analyzed separately, yielding multiple robustness scores. This can provide an observer increased insight into which aspects of a system are non-robust and which perturbations this is encountered. In engineering, such information can be the first step in taking remedial measures to improve robustness in key areas.

However, a key disadvantage of this approach is that the evaluation function, $D(\cdot)$, lacks generality and may be difficult to apply in engineering. For example, it assumes:

1. A nominal operation condition is known (so that $f_a(0)$ can be evaluated)
2. The nominal operating condition is nonzero (to avoid dividing by zero)
3. Each function is characterized by a single performance function, $f_a(\cdot)$
4. Increasing values of the performance metric $f_a(\cdot)$ are preferred

Although some of these assumptions can be relaxed, others are more difficult to avoid in engineering. For example, engineering systems typically are characterized by multiple figures of merit. Even a single function of a system may have multiple figures of merit. Thus, although the biological robustness quantification framework has some advantages and desirable properties, it is unsuitable for direct application in engineering systems development.

2.1.3. **Engineering Robustness**

One can find the term robustness throughout the engineering literature. However, different authors tend to use it differently in different contexts—readers are left to determine its specific meaning in a particular instance. One finds the term robustness in many engineering specialties, including system design and controller design. Each discipline may use the term differently causing some confusion about the exact definition of robustness. Table 3 is an overview of the different definitions used in systems engineering and control engineering.

Table 3: Definition of robustness within different engineering communities.

Systems Engineering	Robust systems should degrade gradually and gracefully in response to component failures, changes in operating environment, and when design loads are exceeded [13].
	Median Performance of the system should be within customer specs [17].
	Performance of the system should be independent of variance [17].
Controls	The control must also be structurally stable or robust, in the sense that regulation of e to zero in the steady-state occurs even in the presence of "small" perturbations of the original system parameters [18].
	Robust control refers to the control of unknown plants with unknown dynamics subject to unknown disturbances [19].

The distinction between *function* and *behavior* becomes an issue when one considers these definitions. In engineering, particularly in engineering design, it is common to consider a function as a transformation of energy, material, or signal that engineers sometimes express using a verb-noun pair [20]. For example, the function of an internal-combustion engine is to “transform energy” (chemical to rotational mechanical). In comparison an engineer would describe the behavior of an internal-combustion engine using one or more performance metrics, such as its efficiency in performing the transformation, the amount of power it can produce, etc. However, this firm distinction between function and behavior do not necessarily hold outside of the engineering disciplines. Although the definition for robustness from the biological sciences community is explicitly in terms of function (Table 2), the robustness measures developed based on this definition relate more closely to behavior in the engineering sense. Thus, the biology community definition is similar to the engineering and manufacturing definitions.

It is interesting to note that although the biological systems robustness measures are in terms of behavioral attributes of the system, they advocate the measurement of robustness on a function-by-function basis [11]. Moreover, one obtains a different robustness score for each function of the system. This is a departure from the other communities surveyed, which do not generally discuss the possibility that one function or aspect of a system could be robust while another one is not.

Ultimately, all communities agree that for a system to be robust it must maintain its effectiveness—in functionality or behavioral performance—under uncertainty. The uncertainty may be caused from several different types of perturbations such as the external environment, internal components, or operating input.

2.1.4. **Related Terms**

In order to adequately examine the definitions of robustness; related terms also have to be examined. These related terms include *resilience* and *reliability*. Resilience is defined as, "capacity of a system to react to an unpredictable perturbation in its environment and to come back to a nominal functioning state" [21]. Another definition presented for resilience is, "A resilient system can handle a wide range of contexts and can be adapted to other situations" [22]. The key difference between definitions for robustness resilience is that a resilient system should handle perturbations that are unpredictable or unanticipated; definitions for robustness tend to encompass only anticipated perturbations. Table 4 is a summary of additional characteristics of a resilient system.

Table 4: Characteristics present in a resilient system [21].

1.	Able to react quickly and efficiently to perturbations and threats.
2.	Able to monitor unexpected perturbations and threats.
3.	Able to anticipate environment changes that may impact the system and adjust to maintain function.

A resilient system shares many qualities of a robust system. For example, both resilient and robust systems are expected to react efficiently to perturbations from internal components or the external environment. The two have some contrast in the fact that a robust system is not expected to "monitor" perturbations or threat but simply maintain its anticipated behavior.

An example of system that may be considered robust but not resilient is a Formula 1 race car. The car is set up specifically for each track and anticipated environment. For example a car may be set up for a curvy track with dry, sunny and 90^0F environmental conditions. The car can adjust to “minor” changes in the conditions such as temperature ($\pm 10^0$F), wear on tires, and amount of fuel in car. Using the engineering definition provided in Table 2 the system would be considered robust. In contrast a Formula 1 car may not be considered resilient due to its inability to adapt to unpredictable perturbations, such as oil on a track or loss of a fin. This lack of ability to adjust to major perturbations shows that the F1 car is not resilient but can still be considered robust.

Another term that shares a similar definition to robustness is *reliability*. Reliability is defined as the probability of successful system operation given known uncertainty in system and environment properties [23, 24]. Using this definition, reliability is a quantitative property that provides insight into the success of the system. Similar to robustness, reliability focuses on anticipated uncertainties that affect a system. One generally would consider an unreliable system to be non-robust. However, a system may be considered non-robust even if its probability of failure is within acceptable bounds.

2.2. **Functional Modeling Background**

Functional modeling is used during concept design in order to develop how the product should unction. Modeling the function forces the engineers to focus on what rather than how it is to be achieved. Product function is defined as, “the overall intended function of the product – what it is to do" [25]. Product functions are often a verb with a noun. For example the function of a coffee grinder is to make coffee grounds. Several different methods have been proposed in order to develop functional models for a design. These include function trees, black box models, and EMS function structures. Function trees or hierarchical functional models begin with global device function and develop what functions are needed to accomplish it below. Figure 3 is an example of a hierarchical function model. The advantage of hierarchical functional models is that they provide the designers with a top-down view of the product function.

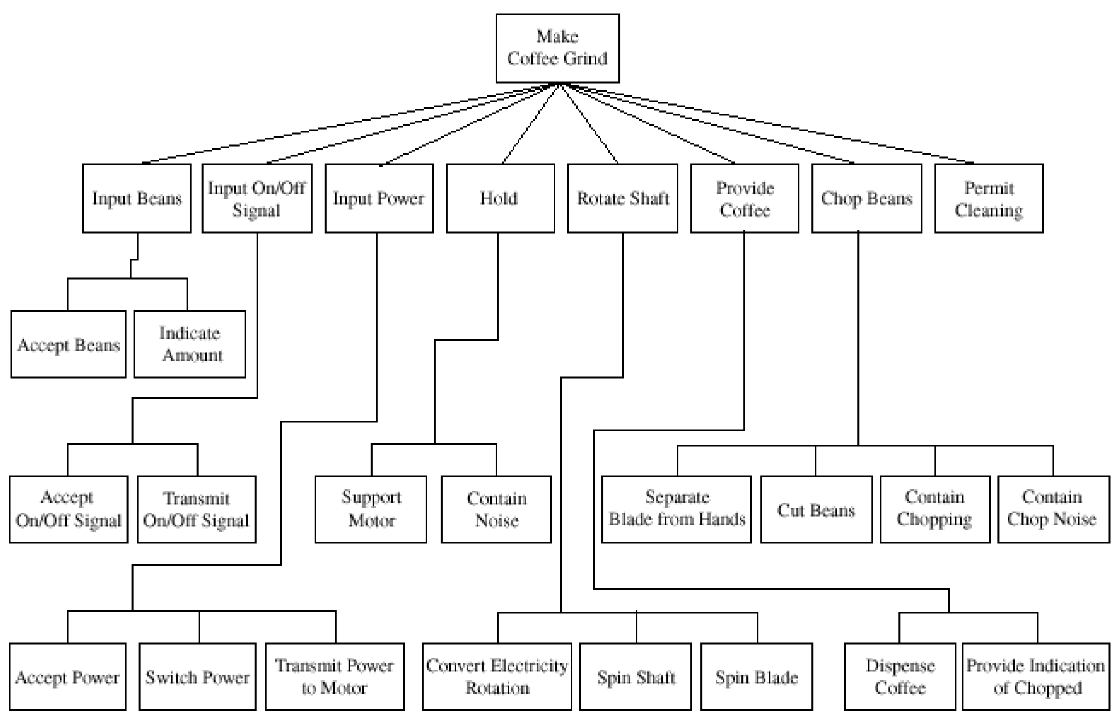

Figure 3: Function tree of coffee mill [25].

Black box models take a different approach from function trees and model the product as a black box with three types of inputs and output. The inputs into the black box model are material, energy, and signal flows. Figure 4 represents the basic black box model of a product.

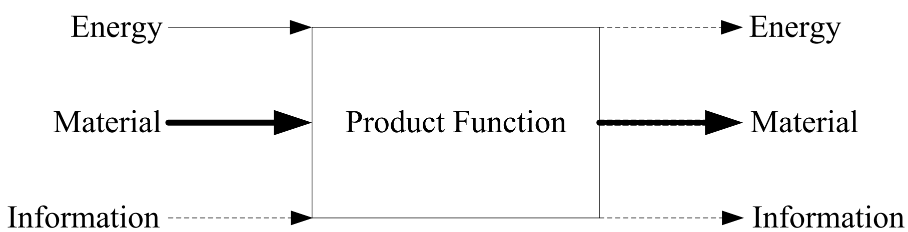

Figure 4: Black box model with energy, material, and information.

A black box model treats the system as an empty box and is only concerned with the energy, material, and information that enters and exits the system. Generating black box models is a crucial first step to developing EMS Function Structure. EMS Function Structures examine the internal interactions within the system. A function structure is defined as, "input-output model that maps energy, material, and signal flows to a transformed and desired state" [25]. The function structure is a systematic approach for modeling system functions. Figure 5 shows the function classes, basic functions, and synonyms used in function structure.

Function Class	Basic Functions	Alternatives (Synonyms)
Channel	Import	Input, Receive, Allow, Form Entrance
	Export	Discharge, Eject, Dispose, Remove
	Transport	Lift, Move, Channel
	Transmit	Conduct, Transfer, Convey
	Guide	Direct, Straighten, Steer
	Stop	Insulate, Protect, Resist, Sheild
Store/Supply	Store	Contain, Collect, Reserve
	Supply	Fill, Provide, Replenish
Connect	Couple	Join, Assemble, Attach
	Mix	Combine, Blend, Add, Pack, Coalesce
Branch	Branch	Divide, Diverge, Switch, Valve
	Filter	Purify, Strain, Filtrate, Percolate, Clear
	Separate	Release, Detach, Disconnect, Disassemble, Subtract
	Remove	Cut, Polish, Sand, Drill, Lathe
	Distribute	Scatter, Disperse, Diffuse, Empty
	Dissipate	Absorb, Dampen, Dispel, Diffuse
Control Magnitude	Actuate	Start, Initiate
	Regulate	Control, Allow/Prevent, Enable/Disable, Limit, Interrupt
	Change	Increase, Decrease, Amplify, Reduce, Normalize, Multiply Scale, Rectify
	Form	Compact, Crush, Shape
Convert	Convert	Transform, Liquefy, Solidify, Gyrate, Evaporate, Condense Integrate, Differentiate, Process
Support	Stabilize	Steady
	Secure	Attach, Mount, Lock, Fasten, Hold
	Position	Orient
	Translate	
	Rotate	Turn, Spin
	Allow DOF	Constrain, Unsecure, Unlock
Signal	Sense	Perceive, Recognize, Discern, Check, Locate, Verify
	Indicate	Mark
	Display	
	Measure	Calculate, Compare, Count

Figure 5: Function classes, basic functions, and synonyms used in EMS functions structures [26].

The basic functions are used to describe the energy, mass, and signal flows through the product. Developed guidelines validate the developed EMS Function Structure [25]. One of the main guidelines for developing the EMS Function Structures is conservation of mass and energy. Function modeling specifically function trees, black box models, and EMS Function modeling is a crucial component of the approach developed in Section 5.

2.3. **Literature Review Conclusion**

The definitions of robustness examined in the previous sections have several similarities. Key components of robustness include maintaining function and uncertainty in internal and external properties. Functional modeling is introduced in order to examine the concept of robustness on a per function basis. For the purposes of this thesis robustness in Systems Engineering will be defined as a property that allows a system to maintain its functions against anticipated internal and external perturbations [13, 21, 27]. A general definition of Systems Engineering robustness opens the door to examine possible quantitative measures of robustness such as variance.

3. QUANTITATIVE MEASURE OF COMPLEX ENGINEERING ROBUSTNESS

In order to accomplish several basic system engineering operations a quantitative measure of robustness is required. This leads to important questions: Is a quantitative measure of robustness needed in systems engineering? Is variance an adequate measure of robustness or is a new metric required? In this section we examine variance as an inverse measure of robustness and its application of robustness on a per function basis.

Variance is a good measure for manufacturing robustness but its extension to complex engineering poses problems. Another approach is examined to include the designer's preference of robustness into the design process without explicitly measuring it. Utility theory is a framework for making decisions. In this section Utility Theory and is ability to express the designer's preferences are also examined.

3.1. Complications of Using Variance as an Inverse Measure of Robustness

A system with low variance in its response is considered robust in Robust Design [28]. Following the definition in Section 2.3 variance would need to be measured on a per function basis. Consider a design problem with four functions. Four separate figure of merit variances would need to be measured but combining the variances becomes difficult. Is each function equivalent in importance or is one more important than the others? How small does each variance need to be in order to ensure robust functions? If each figure of merit is in different units how can their variances be combined? Variance works well in terms of manufacturing robustness but is limited in its application to systems engineering. Developing a general measure of robustness for systems engineering may not be the solution. Another approach is to not measure robustness but to allow the designer to elicit their preferences into the design process.

3.2. Utility Theory Background

Utility Theory is a mathematically rigorous framework for making decisions under uncertainty. Introduced by von Neumann and Morgenstern, the theory is based on several axioms that assert what it means to be “rational” [29]. These axioms imply a means for formalizing one’s preferences mathematically (via what is called a *utility function*) and a procedure for making decisions (choose the option that maximizes the expected value of utility). Although originally formulated for single-objective problems, it has been extended to the case of multiple objectives [30]. Both the single- and multiple-objective formulations have been investigated in the engineering design research literature [31-35].

Let z denote some figure of merit (often referred to in the literature as an *attribute*) that is uncertain and modeled probabilistically. A utility function over z is denoted $u(z)$. According to utility theory, a rational decision maker seeks to maximize the expected value of utility shown in Equation (3):

$$z^* = \arg\max_{z \in Z} E[u(z)] \tag{3}$$

where Z is the feasible set for values of z. Here it is understood that the figure of merit is a function of some underlying design variables and impacted by some uncertainty that may be internal or external to the system. Let x denote the designable variables and ε denote the uncertain variables. Thus, one has $= f(x, \varepsilon)$, which parallels the mapping $y = f(x, \varepsilon)$ in Robust Design. Typically in an engineering design problem, one searches the space of designable variables directly. However, this discussion will be limited to the attribute space (i.e., the z-space) without loss of generality.

A utility function conveys complete information needed to understand a decision maker's preferences. This includes their risk attitude—how they deal with risk due to uncertainty in a decision problem. To explain clearly the meaning of risk attitude, it is helpful to introduce the concept of a *lottery*. Let $\langle a, b, \pi \rangle$ denote a chance event in which one wins a units of a variable with probability π and b units with probability $1 - \pi$. An individual is considered *risk averse* if they prefer the expected consequence of a non-degenerate lottery to that lottery [30, 36]. A non-degenerate lottery is one in which no outcome occurs with probability one. For example, a risk averse individual would prefer to take 50 cents with certainty rather than engage in the lottery $\langle \$1, \$0, 0.5 \rangle$ (i.e., a lottery that pays \$1 with probability 0.5 and \$0 otherwise—and therefore has an expected value of fifty cents). A consequence of this definition is that one's utility for the expected value of the lottery is larger than the expected utility for the lottery. Let $\tilde{z}$ denote a non-degenerate lottery. Equation (4) demonstrates this for a risk averter:

$$u(E[\tilde{z}]) > E[u(\tilde{z})] \tag{4}$$

Another important concept is the *certainty equivalent* of a lottery, which is the amount one would take to be indifferent between that amount and the lottery. Let $\tilde{z}$ denote a lottery and $\hat{z}$ denote its certainty equivalent. Equation (5) denotes the utility at the certainty equivalent. Equation (5) can be written as the inverse shown in Equation (6):

$$u(\hat{z}) = E[u(\tilde{z})] \tag{5}$$

$$\hat{z} = u^{-1}(E[u(\tilde{z})]) \tag{6}$$

where $u^{-1}(\cdot)$ is the inverse of the utility function.

This leads to two more important concepts:

- An individual's *risk premium* for lottery $\tilde{z}$ is the expected value of the lottery minus the individual's certainty equivalent for that lottery. This is shown in Equation (7).

$$RP(\tilde{z}) = E[\tilde{z}] - \hat{z} = E[\tilde{z}] - u^{-1}(E[\tilde{z}]) \tag{7}$$

- An individual's *insurance premium* for lottery is the negative of that individual's certainty equivalent for that lottery: $IP(\tilde{z}) = -\hat{z}$.

The risk premium represents the amount by which the expected return on the risky option (the lottery) must exceed the value of a certain alternative (the certainty equivalent) for a decision maker to conclude the risk is worth taking. Thus, larger risk premiums imply a larger degree of risk aversion. On the other hand, the insurance premium is the amount a decision maker would be willing to pay to get rid of the risky option (the lottery). Larger insurance premiums are a sign that the decision maker is unfavorable to the level of risk in an alternative.

It is possible to construct a useful measure of risk aversion that can be easier to interpret than an individual's risk premium or insurance premium. An individual's local risk aversion is defined as Equation (8) [36].

$$r(z) = \begin{cases} -\dfrac{u''(z)}{u'(z)} & \text{for } u'(z) \geq 0 \\ \dfrac{u''(z)}{u'(z)} & \text{for } u'(z) < 0 \end{cases} \tag{8}$$

This presumes that an individual's utility function is twice differentiable. Notice the sign difference in the definition depending on the value ordering induced by $u(\cdot)$. This is to preserve the semantics of the local risk aversion such that an individual is a risk averter if and only if $r(z) > 0$. Larger values of $r(\cdot)$ mean one is more risk averse. Behaviorally, this means one would give up more in order to avoid the risk.

3.3. Quantitative Measure of Robustness Conclusion

Developing a general quantitative measure of robustness is extremely difficult. Variance has been used as an inverse measure of robustness and while it may be affective in a manufacturing sense it is difficult to extend to systems engineering. One of the major issues of applying variance as a measure of robustness is combining the variance terms in systems that have multiple functions. From this it is determined that robustness should not be explicitly measured. A possible solution is to allow the designer to elicit their preferences within a utility function. Another possible solution is to extend Robust Design or quality engineering to systems engineering.

4. ROBUST DESIGN AND SYSTEMS ENGINEERING

Robust Design also known as quality engineering must be examined when discussing robustness. Robust Design is examined because it is a design method that tries to maintain performance without eliminating the variance or uncertainty in the system. Can we extend Robust Design to engineer robust systems? In this chapter background on how Robust Design was developed from the perspective of quality loss is examined. Originally developed with the goal of developing quality products fast and with low cost Robust Design's purpose has expanded since its inception [6]. The foundations of Robust Design were originally developed by Taguchi in order to decrease the cost to manufacture a product and still maintain high quality in the product. Early quality loss engineers supported the use of a quadratic loss function. The quadratic quality loss function has been manipulated into a mean-variance function. The mean-variance approach is often taken as a design methodology that develops robust systems.

Supporters of Robust Design claim that it is a simple yet effective way to develop robust systems. The assumptions and preferences placed upon the designer when using his approach are unexamined within the Robust Design field. Utility theory is employed within this chapter to investigate the preferences placed upon the designer and whether or not they are realistic.

4.1. **Robust Design Background**

Robust Design, also referred to as Quality Engineering, is an approach used by engineers to design systems and products. The foundation of the method is credited to Genichi Taguchi. Robust design aims to eliminate the sensitivity a product has to uncontrollable factors such as manufacturing variability and environmental conditions [7, 37-40]. As put in one text, "a product or process is said to be robust when it is insensitive to the effects of sources of variability, even though the sources themselves have not been eliminated" [41].

Fundamentally, Robust Design is a parametric optimization scheme in which one seeks a solution that is insensitive to small to moderate scale perturbations to the operating point of the system. Figure 6 is an illustration of this concept. Function $F(x)$ represents an objective function that one wishes to maximize. Mathematical optimization aims to find the global maxima while Robust Design aims to find the robust optimum. If uncertainty is present in x a system designed at the global optimum may fall into the valleys present on each side of the optimum. The Robust Design optimum will be more consistent in performance if there is uncertainty in the value of variable x (e.g., due to manufacturing variability).

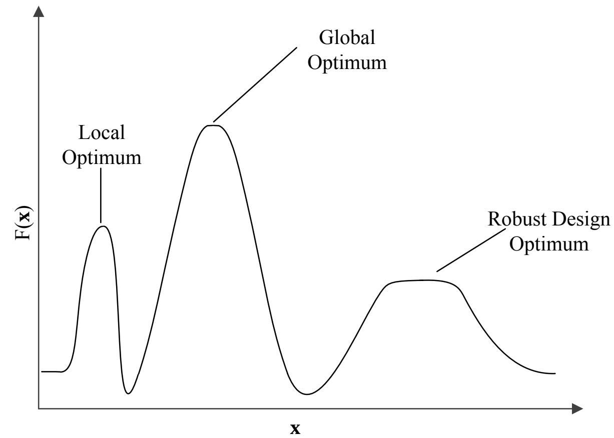

Figure 6: Performance of a system may have global and robust optimal.

The parameters of a system can be divided into control, noise, and signal factors. Signal factors represent the range of possible configurations that may occur during system operation. Noise factors represent parameters that are expensive or unable to be controlled and that are subject to uncertainty. Control factors are the design variables and are adjusted to obtain the optimal robust solution. In this context, a robust system or product is one that is insensitive to variability in noise and control factors [42]. Type 1 Robust Design focuses on minimizing variation caused by the noise or uncontrollable factors. Type 2 Robust Design focuses on minimizing the variation caused by the design variables or control factors [43]. Figure 7 is an illustration of the difference between Type 1 and Type 2 Robust Design. Although the ultimate aims are the same—design a system for which the response is insensitive to the uncertainty—one may benefit from different techniques depending on the type of problem.

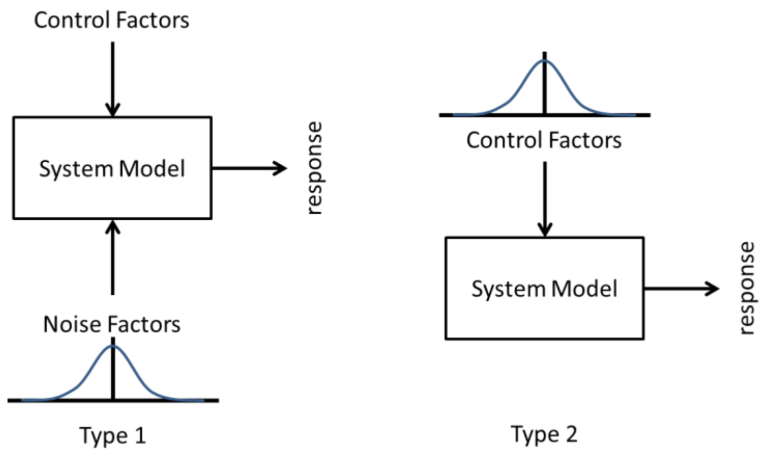

Figure 7: Robust design type comparison [43].

Robust design methods are based upon the quadratic loss function which is a target-seeking equation. The quadratic loss function is the squared difference between the response and the target value. Equation (9) shows the basic form of the quadratic loss function,

$$L(y,t) = (y - t)^2 \tag{9}$$

where y is the system response and t is the desired target value. Generating a response that is away from the target is undesirable because it equates to loss of quality in a system. The response, y, is normally a function of noise factors, z, and control factors, x. As stated before noise factors are uncontrollable and are modeled using probability density functions causing both the response and loss values to be random variables. Minimizing the expected loss subject to any constrains is the overall objective and is shown in Equation (10).

$$x^* = arg \min_x E[(f(x,z) - t)^2]$$

subject to (10)

$$g_i(x,z) \leq 0 \; i = 1, \ldots, n$$

The equality constraints shown are rarely used in the Robust Design literature. Equation (11) shows the expected loss function rewritten in terms of mean and variance,

$$E[(y - t)^2] = \sigma_y^2 + \left(\mu_y - t\right)^2 \qquad (11)$$

where μ_y and σ_y are, respectively, the mean and variance of response y. Examining the equation it can be deduced that the objective of Robust Design is to minimize variability and deviation from target simultaneously. Additional freedom has been given to designers by introducing a reweighting term. Equation (12) shows the quadratic loss function with the additional reweight term [44],

$$E[L] = w\sigma_y^2 + (1 - w)\left(\mu_y - t\right)^2 \qquad (12)$$

where w is restricted to values between 0 and 1.

Another variation of the quadratic loss function is the square root of individual terms within the function. Taking this approach allows the equation to be in the units of the response variable. Equation (13) is the quadratic loss function taking the square root of individual terms.

$$E[\tilde{L}] = \sigma_y + \left(\mu_y - t\right) \qquad (13)$$

This is not the same as taking the square root of the entire quadratic loss function.

Use of the quadratic loss function has led people to focus on mean and variance as key figures of merit in a Robust Design problem with variance serving as a quantification of robustness. In turn, some formulate a Robust Design problem as a multi-objective optimization problem in which mean response and response variability are the objectives to be optimized. Letting $\mu_{y-t} = \mu_y - t$, one can formulate this as shown in equation (14):

$$\min_{x}\left[\mu_{y-t}, \sigma_y\right]$$

$$\text{subject to} \tag{14}$$

$$g_i(x, z) \leq 0 \; i = 1, \dots, n$$

where it is understood that μ_{y-t} and σ_y depend on x. Potential advantages of this approach include that it admits the use of multi-objective optimization techniques and that it provides flexibility to weigh the different factors (mean and variance) differently.

Analogous loss functions and problem formulations are possible for situations in which one seeks to maximize or minimize the response variable, y, rather than achieve a particular target value. This goes back to Taguchi, who proposed different techniques for each of the situations [7]. Extensions also exist for Robust Design with multiple responses (i.e., y is a vector) [45, 46].

4.2. Utility Based Critique of Robust Design

It is constructive to consider Robust Design from a utility-theoretic perspective. The quadratic loss function shown in Equation (9) defines one's preferences under a Robust Design scheme. To formulate this in a manner consistent with utility theory, let $u(y,t) = -L(y,t) = -(y-t)^2$, where y is the response and t is the target value for the response. To keep the results general to any targeted value, let $z = y - t$. This gives us the utility function shown in Equation (15).

$$u(z) = -L(z) = -z^2 \tag{15}$$

According to utility theory, a rational decision maker seeks to maximize expected utility. The max expected utility of the quadratic utility function is shown in Equation (16).

$$\max_z E[u(z)] = -E[z^2] \tag{16}$$

Equation (16) can be rewritten as Equation (17) which is equivalent to (10).

$$\max_z -\sigma_z^2 - E^2[z] \tag{17}$$

It is straightforward to see that this is a universally risk averse preference structure. It is known that a decision maker is risk averting over the entire decision domain if and only if the second derivative of their utility function is negative [30]. The second derivative of the utility function shown in Equation (18) is negative.

$$\frac{d^2u(z)}{dz^2} = -2 \tag{18}$$

Thus, this is a risk averse preference structure.

One also can analyze the risk attitude of this preference structure through the local risk aversion function. Since $u'(z) = -2z$ and $u''(z) = -2$, the local risk aversion function can be determined from Equation (8) giving Equation (19).

$$(z) = \begin{cases} -\frac{1}{z} & \text{for } z \leq 0 \\ \frac{1}{z} & \text{for } z > 0 \end{cases} \tag{19}$$

The local risk aversion function (Equation (19)) is plotted in Figure 8.

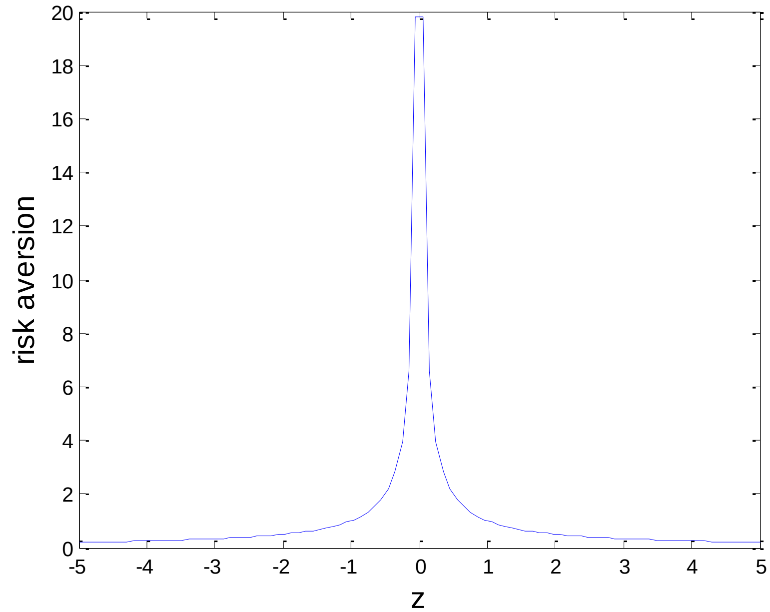

Figure 8: Local risk aversion as a function of difference between response and target value.

The function has several key features to consider:

- It is positive for all z. This means it is universally risk averting.
- Risk aversion increases as one approaches the target value (recall that $z = 0$ when $y = t$).
- It has a discontinuity at the origin (i.e., when $y = t$)

It is unclear whether a risk aversion relationship of this nature is reasonable. For $z \leq 0$, risk aversion is an increasing function—i.e., one becomes more risk averse as the target value is approached. This is called Increasing Absolute Risk Aversion (IARA) and has some questionable behavioral implications. For example, suppose a decision maker who wishes to maximize wealth and is IARA. Suppose further that the decision maker is indifferent between the lottery $\langle \$100, \$200, 0.5 \rangle$ and the certain option $\$125$. The IARA implies that this same decision maker prefers the certain option $\$1125$ to the lottery

$\langle \$1100, \$1200, 0.5\rangle$ (i.e., the certainty equivalent is less than \$1125). Both lotteries have the form $\langle x-h, x+h, 0.5\rangle$, for some reference point x and perturbation $h>0$, and the perturbation size in both lotteries is identical ($h = \$50$). However, the decision maker becomes more risk averse at larger values of money and therefore has a lower certainty equivalent and larger risk premium.

To understand how this risk attitude can seem questionable, consider that the second lottery is the same as saying that the decision maker will receive \$1000 with certainty and engage in the lottery from Scenario 1, $\langle \$100, \$200, 0.5\rangle$. Why should the extra certain earnings make one more risk averse? The example is shown in Table 5 and 6.

Table 5: Monetary example of IARA attitude and its implications.

Scenario	$\boldsymbol{x}$	$\boldsymbol{h}$	Lottery $\langle \boldsymbol{x}-\boldsymbol{h}, \boldsymbol{x}+\boldsymbol{h}, \mathbf{0.5}\rangle$	Expected Value	Certainty Equivalent	Risk Premium
1	\$150	\$50	$\langle \$100, \$200, 0.5\rangle$	\$150 $(=x)$	\$125	\$25
2	\$1150	\$50	$\langle \$1100, \$1200, 0.5\rangle$	\$1150 $(=x)$	<\$1125	>\$25

Table 6: Equivalent Scenario 2 written in terms of Scenario 1.

Original Scenario 2 Lottery	Equivalent Scenario 2 Lottery in terms of Scenario 1 lottery
$\langle \$1100, \$1200, 0.5\rangle$	$\$1000 + \langle \$100, \$200, 0.5\rangle$

This argument also would seem to hold for the target achievement case represented by Robust Design. Although the use of negative values can be unintuitive at first, this is exactly the same as the monetary example: Scenario 2 represents a situation in which the decision maker is uniformly better off than in Scenario 1. To be more concrete, suppose a decision maker is concerned with variations in manufacturing of mechanical parts and z represents the difference between the actual manufactured dimension of a part and its nominal dimension (in units of mm). Both scenarios involve identical variability in the manufacturing outcome, but Scenario 1 has a worse mean outcome compared to Scenario 2 (by 2 mm). Even though the worst-case outcome in Scenario 2 is better than the best-case outcome from Scenario 1, an IARA decision maker places a larger risk premium on Scenario 2. This means the decision maker would pay more to rid themselves of the gamble represented in Scenario 2 than that of Scenario 1—an unintuitive and unlikely situation. The example is described in Table 7.

Table 7: Engineering example of IARA attitude and it implications.

Scenario	$\boldsymbol{x}$	$\boldsymbol{h}$	Lottery $\langle \boldsymbol{x-h, x+h, 0.5} \rangle$	Expected Value	Certainty Equivalent	Risk Premium
1	-3	0.5	$\langle -3.5, -2.5, 0.5 \rangle$	-3 ($= x$)	-3.3	0.3
2	-1	0.5	$\langle -1.5, -0.5, 0.5 \rangle$	-1 ($= x$)	< -1.3	> 0.3

An analogous argument can be made for values of $z \geq 0$. Note that in this case, risk aversion increases as one approaches $z = 0$ from the right (i.e., as z decreases).

One important remark is that this analysis highlights how a decision maker following a Robust Design framework places more importance on variability as the target value is approached. This is not evident by inspecting the objective function typically employed in Robust Design. Moreover, simple inspection of the objective function based on expected quadratic loss would lead many to conclude that variability and target-achievement are given equal weight. This unintuitive result represents a significant potential pitfall of using typical Robust Design techniques. Target-achieving preferences would appear to place some restrictions on the form of available utility functions and these restrictions may be problematic.

4.3. Limitations on Target-Achieving Preferences

Let t denote some target value an individual wishes to achieve and let y denote the actual value of the corresponding figure of merit. From an engineering standpoint, y is a function of variables over which the designer has decision-making authority. For notational convenience, let $z = y - t$ represent the difference between the actual and targeted values. The individual's decision objective will be to minimize this difference. Thus, the person's utility function will be a function of z. Let $u_T(z)$ denote this utility function and assume it is twice differentiable.

The following necessary conditions exist on this utility function:

1. $u_T(z)$ is increasing for $z < 0$. Alternatively stated: $u_T'(z) > 0$ for $z < 0$
2. $u_T(z)$ is decreasing for $z > 0$. Alternatively stated: $u_T'(z) < 0$ for $z > 0$
3. The point $z = 0$ is a stationary point. Alternatively stated: $u_T'(z = 0) = 0$

These conditions imply the following results:

1. The point $z = 0$ is a local maximum. Alternatively stated: $u_T''(z = 0) < 0$
2. The point $z = 0$ is a global maximum.

These results are straightforward. Intuitively, since $u_T(z)$ is increasing until $z = 0$, has a slope of zero at that point, and then is decreasing thereafter it stands to reason that $z = 0$ is the global maximum of $u_T(z)$. A global maximum also is a local maximum. The problem with a target-seeking utility function lies in the implications this has for risk attitude. Recall that an individual's local risk aversion is defined as Equation (8).

The problem with target seeking is evident in the denominators of $r_T(z)$—local risk aversion is undefined at the target value. This is because $z = 0$ is a stationary point (zero slope). If $u_T''(z)$ also goes to zero at $z = 0$, it may be possible to evaluate the fraction using a limit. That possibility aside, this is a serious problem as it suggests that a decision maker seeking to achieve a specific target value will necessarily become infinitely risk averse as the actual value approaches the target value .

It is unclear whether this is reasonable behavior in the immediate neighborhood of a target value. However, as discussed previously, increasing absolute risk aversion is an unrealistic preference structure in many situations.

It is possible to avoid the problem of increasing risk aversion (and infinite risk aversion) by sacrificing the stationary point assumption. Moreover, by allowing a discontinuity in $u_T(\cdot)$ and $z = 0$, it is possible to approach the target with a decreasing or constant risk aversion. The cost of this of course is that the solution becomes difficult to define mathematically. Practically speaking, it may be more important to preserve the stationary point than the desired risk function behavior.

Note that this analysis assumed very little about the form of $u_T(z)$. For example, we did not assume it was based on the quadratic loss function or any other particular function. Thus, these results are very general and apply to any target-seeking preference structure.

4.4. **Robust Design and System Engineering Conclusion**

Robust Design and target seeking design methods impose preferences upon the designer which may not accurately model the designer's preferences. A design method should provide the designer with the ability to express their preferences in a mathematical framework. In addition a method should incorporate robustness into its steps without directly measuring robustness. This is due to the fact that robustness is difficult to measure and may not be applicable to every system. Due to these factors Robust Design is not extended to systems engineering. Instead a utility-based analysis for increased robustness described in the next section accomplishes these aspects.

5. UTILITY-BASED ANALYSIS FOR INCREASED SYSTEM ROBUSTNESS

Section 4 shows the preferences placed on the engineering when using Robust Design to engineer a system. The imposed preferences may not match that of the engineers and alternatives need to be examined in order to design robust systems. In this section a utility-based approach for increased system robustness is presented. The approach allows for the designer to elicit their own preferences within the model. The eleven-step approach combines several key characteristics in order to develop a robust system. Key characteristics include insight into important perturbations and where the designer should spend additional resources in order to increase the robustness of the system. The key components include functional modeling, utility theory, and a sensitivity analysis. Each component is important in order to achieve the desirable characteristics.

5.1. **Background**

A utility-based design method provides several favorable characteristics when designing a system. Utility theory is a normative approach for comparing designs with multiple attributes under uncertainty [9]. The designer formulates a utility function that incorporates their preferences without imposing preferences on the designer. This characteristic allows the designer to develop the importance of maintaining function based upon their own preferences. The designer uses the developed utility function to rank order possible designs in order to select the design that best meets their preferences. The purpose of the utility-based approach is to provide valuable information to the designer on where to spend additional time and money in order to develop a system with higher robustness. The approach achieves this through providing information on which perturbations should be included and by forcing the designers to examine the sensitivity of the system/subsystem figures of merit. Focusing on each function is key when trying to develop a system with higher robustness.

The approach is to be started after the concept design stage has been completed. Functional models may have been created within the concept design stage but should be recreated once the general system design has been developed. Functional models should be created for each subsystem function; this requires the system to be broken down into subsystems and for the designer(s) to discuss each subsystems function. In order to measure the system's and subsystems' ability to maintain function, figures of merit for each must be determined. More than one figure of merit may measure a single function.

Elicitation of the system level utility function should be done using appropriate methods and assumptions.

5.2. **Utility-Based Approach for Increased System Robustness Steps**

The utility-based analysis with focus on robustness consists of eleven-steps:

Step 1: Model function(s) of the system and subsystems (Energy, Mass, and Signal (EMS) Function Structure, Function Tree, black box model, etc.)[25].

Step 2: Determine figure(s) of merit for system and subsystems functions.

Step 3: For each subsystem function, identify perturbations that impact functional performance. Model anticipated internal and external perturbations with appropriate probability density functions.

Step 4: At the system level, model any perturbations that affect the system as a whole. (Some perturbations may not impact individual functions, but may impact the system as a whole).

Step 5: Model each function's figure(s) of merit in terms of identified perturbations and relevant variables.

Step 6: Elicit system level utility function. Decisions With Multiple Objectives provides background on elicitation of utility functions. Make appropriate simplifying assumptions.

Step 7: Use optimization techniques to compute the maximum expected utility of the system (using appropriate sampling of perturbations).

Step 8: Perform sensitivity analysis on figures of merit with respect to anticipated internal and external perturbations. This paper performs a one at a time sensitivity analysis of the uncertain parameters.

Step 9: Using the information gained from the sensitivity analysis, determine which system/subsystem functions are most susceptible to loss of function. Examine the range at which the system is able to perform each function. Examine figures of merit that have a high rate of change when a parameter is changed.

Step 10: For susceptible function(s), examine the use of different technologies, redundancy, and/or modularity [47].

Step 11: Repeat steps 1-10 until system and subsystems adequately maintain function in terms of anticipated internal and external perturbations.

5.3 Comparison of New Utility-Based Analysis To Standard

In this Section we compare the utility-based analysis to the standard utility-based design approach utilized in engineering. Method for the Evaluation of Design Alternatives (MEDA) was proposed by Deborah L. Thurston and is used as the standard utility-based design approach [34]. MEDA contained six major steps for implementing the design process. These six steps are matched with their closest counterpart in the utility-based analysis for increased system robustness. Table 8 shows a comparison of the two methods.

Table 8: Comparison of utility-based design methods.

Utility-Based Analysis for Increased System Robustness	**Methodology for the Evaluation of Design Alternatives (MEDA)**	**Differences Comments**
Step 1: Model function(s) of the system and subsystems.	Step 1: Define design problem in terms of function.	Utility-based analysis relies on explicitly modeling the functions using EMS Function Structures, Function Trees, and Black Box Models.
Step 2: Determine figure(s) of merit for system and subsystems functions.	Step 2: Distinguish between design criteria and design attributes.	Both methods require definition of design criteria. The utility-based analysis links specific FOM(design attributes) with subsystems.
Step 3: Determine relevant perturbations for each subsystem.		Step is not specifically stated by MEDA but would be required to evaluate utility function.
	Step 3: Define acceptable attribute ranges.	This step is not explicitly stated in the utility-based analysis for increased system robustness but must be completed in order to generate single attribute utility functions.
Step 4: Model any perturbations at the system level.		Step is not specifically stated by MEDA but would be required to evaluate utility function.
Step 5: Model each function's figure(s) of merit in terms of identified perturbations and relevant variables.		Step is not specifically stated by MEDA but would be required to evaluate utility function

Table 8: Continued.

Utility-Based Analysis for Increased System Robustness	Methodology for the Evaluation of Design Alternatives (MEDA)	Differences/ Comments
Step 6: Elicit system level utility function.	Step 4: Determine the worth imparted to the designer over the attribute range Step 5: Determine the multiattribute utility function	Each method requires the elicitation of a system (multiattribute) utility function.
Step 7: Use optimization techniques to compute maximum anticipated utility of the system.	Step 6: Determine the tradeoffs between attributes which would be beneficial to the designer	Both methods use a type of optimization to find best design solution based upon utility.
Step 8: Perform sensitivity analysis on figures of merit.		Both methods suggest using a type of sensitivity analysis. MEDA uses sensitivity to determine how much better an alternative is to another while the utility-based analysis for increased system robustness uses it to compare each subsystems ability to maintain their function.
Step 9: Determine which system/subsystem functions are most susceptible to loss of function.		MEDA does not examine alternatives based upon system/subsystem function but upon important design attributes.
Step 10: For susceptible function(s), examine the use of different technologies, redundancy, and/or modularity.		Designer must examine possible tradeoffs for increased cost for increased performance (robustness).

From the previous tables it can be seen that both methods are similar in their approach. Both methods required the elicitation of a multiattribute utility function. In addition both approaches required the designer to determine figure(s) of merits or design attributes for the design. The utility-based analysis takes several additional steps in order to develop systems and subsystems that are robust or able to maintain their functions. For example in step 1 MEDA simply defines the system in terms of function while the utility-bases analysis takes it a step further and forces the designer to produce functional models at the system and subsystem level. This provides the designer with additional information about relevant perturbations for the system as a whole and for each subsystem being designed. MEDA explicitly forces the designer to determine the acceptable ranges of attributes but this required in order to properly elicit single attribute utility functions needed to develop the multiattribute utility function. Both approaches use sensitivity analysis to examine the possible design solutions. MEDA uses sensitivity analysis to how much better an alternative is compared to another alternative while the utility-based analysis uses sensitivity analysis to determine which system/subsystem functions are susceptible to perturbations. The utility-based analysis is very similar to the approach developed by Thurston with additional steps focusing on improving system robustness.

6. CASE STUDY OF ENTRY, DESCENT, AND LANDING OF MARS ROVER

In order to demonstrate the steps of the utility-based approach for higher system robustness a case study is performed in this section. The case study demonstrates the advantages of the steps described in Section 4. The case study examines the entry, descent, and landing (EDL) of a Mars rover. The EDL system possesses several advantages as a case study: system is comprised of several subsystems, contains complex dynamics, and contains several internal and external perturbations. The EDL system is broken down into four stages: entry, parachute descent, powered descent, and sky crane. Background on each stage is given in further detail within the section. Relevant internal and external perturbations are modeled and a robust system is created using the utility based approach. The steps described in Section 4 are executed following the prescribed procedure.

6.1. **Background of the EDL System**

In order to utilize the utility-based approach developed in Section 5 and complex case study is developed. The Mars Rover Entry, Descend, and Landing or EDL system is broken into four key stages: entry, parachute descent, powered descent, and sky crane. The main objective of the EDL system is to safely land the rover on the surface of Mars. In order to accomplish this goal the rover must land at a safe speed and within a certain distance of the scientific landing zone. The landing sequence is similar to the approach utilized by the NASA Curiosity Rover [1]. A graphical representation of the EDL system is shown in Figure 9.

The EDL system is broken down into subsystems. The subsystems correspond with the four stages of the EDL system. Table 9 shows the system broken down into subsystems functions and objectives. The figures of merit for each subsystem function are also shown. The system and subsystem figures of merit are used within the utility-based approach to determine the total utility of the system and in order to perform the sensitivity analysis. The design variables for the case study include the diameter of the parachute and the amount of rocket fuel carried onboard the EDL. Design constraints for the design problem include: number of rockets, max rocket thrust, crane speed, and rover mass. Table 10 lists the design variables and design constraints when designing the EDL system.

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[1] http://www.nasa.gov/mission_pages/msl/index.html

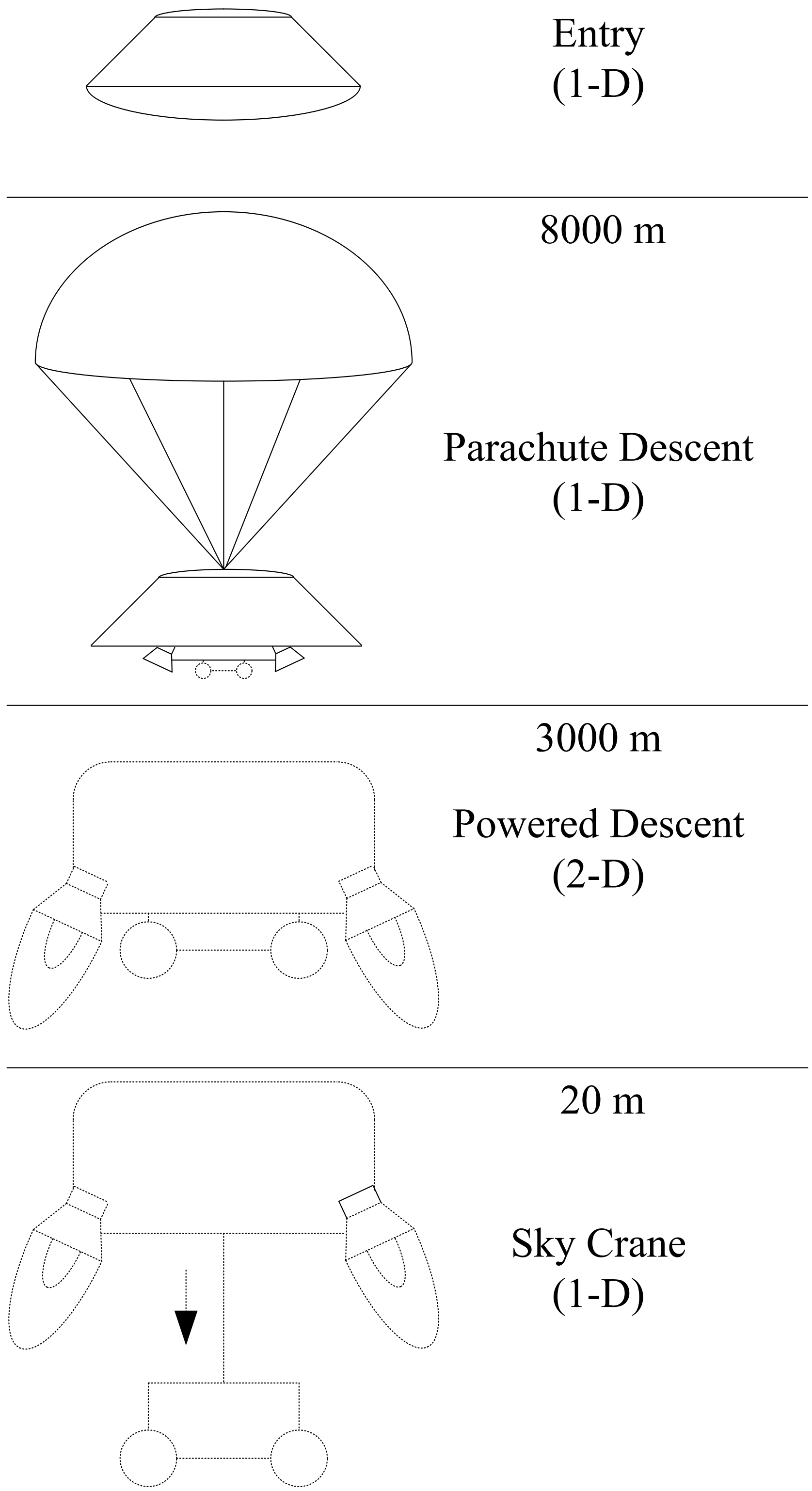

Figure 9: Graphical representation of EDL system.

Table 9: Describes the function, objective, and figure of merit each stage of the EDL system.

	Function	Objective	Figure of Merit
System	Land Rover	maximize success of mission	Landing Safely
		minimize cost	Cost
Subsystems	Protect Rover	Minimize damage to rover during descent	Rover functionality
	Slow Descent	minimize capsule speed at ejection elevation	Capsule Velocity (Parachute Stage)
	Control Descent	minimize y-position overshoot	y-position overshoot
		minimize x-position error	x-position error
	Lower Rover	minimize landing speed	Rover Landing Speed

Table 10: Design variables and constraints of the EDL system.

Design Variables		**LB**	**UB**
	Parachute Diameter	0 m	20 m
	Rocket Fuel Mass	0 kg	500 kg
Constraints		**Value**	**Units**
	Crane Lowering Speed	0.2	m/s
	Rover Mass	775	kg
	Number Rockets	8	
	Max Rocket Thrust	3400	N

6.1.1. **Dynamics of the EDL System**

In order to develop and execute a design study on the Mars rover lander a dynamical model is developed. Figure 10 shows the free body diagram for each stage of the EDL system. The powered descent stage is 2 dimensional while the other stages of the EDL are 1 dimensional.

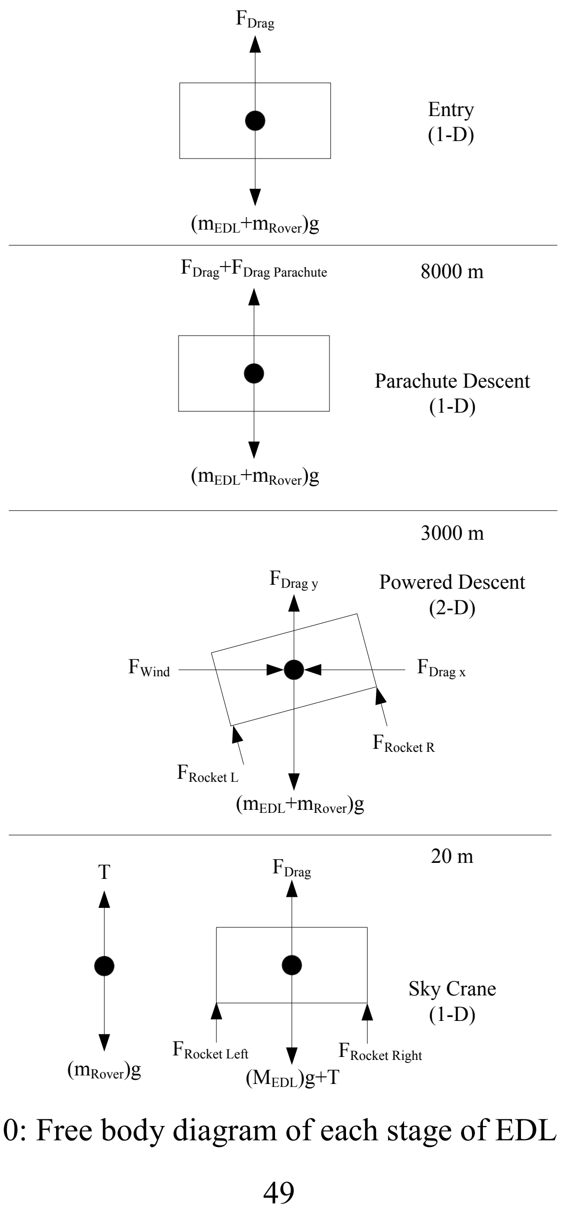

Figure 10: Free body diagram of each stage of EDL system.

The equations of motion are developed for each stage of the Mars rover lander from the free body diagrams. Equations (20) – (26) are the equations of motion for each separate stage of the EDL system.

Entry Stage:

$$(m_{EDL} + m_{Rover})\ddot{y} = F_{drag} - (m_{EDL} + m_{Rover})g \tag{20}$$

Parachute Stage:

$$(m_{EDL} + m_{Rover})\ddot{y} = F_{Drag} + F_{Drag\ Parachute} - (m_{EDL} + m_{Rover})g \tag{21}$$

Powered Descent Stage:

$$\begin{aligned}(m_{EDL} + m_{Rover})\ddot{y} = F_{Rocket_L}\cos(\theta) + F_{Rocket_R}\cos(\theta) + F_{drag_y} \\ - (m_{EDL} + m_{Rover})g\end{aligned} \tag{22}$$

$$\begin{aligned}(m_{EDL} + m_{Rover})\ddot{x} = -F_{Rocket_L}\sin(\theta) + -F_{Rocket_R}\sin(\theta) - F_{drag_x} \\ + F_{Wind}\end{aligned} \tag{23}$$

$$J\ddot{\theta} = F_{Rocket_R}\frac{L_{EDL}}{2} - F_{Rocket_L}\frac{L_{EDL}}{2} \tag{24}$$

Sky Crane Stage:

$$(m_{EDL})\ddot{y} = F_{Drag} - (m_{EDL})g + T + F_{Rocket_L} + F_{Rocket_R} \tag{25}$$

$$(m_{Rover})\ddot{y} = T - (m_{Rover})g \tag{26}$$

The equations of motion for the EDL system are solved using MATLAB. The EDL system needs to be controlled in order to ensure that the rover lands within a certain distance of the objective at a certain velocity.

6.1.2. **Control of the EDL System**

The powered descent and sky crane stages must be controlled in order to keep the system stable. This is accomplished by using separate PID controllers for the powered descent and sky crane stages. The objective of the controller in the powered descent stage is to slow the EDL's descent and adjust x–position. Two controllers are utilized in order to achieve the stage's objectives: attitude control and y–velocity [48]. The x–position of the EDL is not directly controlled but adjusted for by controlling the attitude of the lander. The desired θ–position is dependent upon both the x–position and x–velocity. Equation (27) shows how θ_d is determined,

$$\theta_d = \tan^{-1}(-(C_1(x_d - x) + C_2(\dot{x}_d - \dot{x}))) \tag{27}$$

where C_1, C_2, x_d,and $\dot{x}_d$ are position constant, velocity constant, x–position desired, and x–velocity desired respectively. To ensure that the EDL system does not spin out of control θ_d is limited to $\pm$.2 radians. The control force from the attitude control is shown in Equation (28) and (29).

$$e_\theta = \theta_d - \theta \tag{28}$$

$$F_{Rocket_{R_\theta}} = -F_{Rocket_{L_\theta}} = \frac{k_{p_\theta} e_\theta + k_{d_\theta} \frac{de_\theta}{dt}}{2} \tag{29}$$

In order to determine the total control force during the descent stage the y–velocity must also be considered. The control force from the speed control is shown in Equation (30) and (31).

$$e_{\dot{y}} = \dot{y}_d - \dot{y} \tag{30}$$

$$F_{Rocket_{R_{\dot{y}}}} = F_{Rocket_{L_{\dot{y}}}} = \frac{k_{p_{\dot{y}}} e_{\dot{y}} + k_{i_{\dot{y}}} \int e_{\dot{y}} dt + k_{d_{\dot{y}}} \frac{de_{\dot{y}}}{dt}}{2} \tag{31}$$

The total control during the descent stage is the sum of the attitude control and speed control. Equation (32) and (33) shows the total control force for the EDL system during the descent stage.

$$F_{Rocket_R} = F_{Rocket_{R_{\dot{y}}}} + F_{Rocket_{R_\theta}} \tag{32}$$

$$F_{Rocket_L} = F_{Rocket_{L_{\dot{y}}}} + F_{Rocket_{L_\theta}} \tag{33}$$

The objective of the PID controller during the sky crane stage of the EDL system is to maintain altitude. The control force is determined the same way as the attitude and speed control during the controlled descent stage. Equations (34) and (35) show the control force during the sky crane stage.

$$e_y = y_d - y \tag{34}$$

$$F_{Rocket_R} = F_{Rocket_L} = \frac{k_{p_y} e_y + k_{i_y} \int e_y dt + k_{d_y} \frac{de_y}{dt}}{2} \tag{35}$$

The combination of the attitude, speed, and altitude control allows the EDL system lands the rover safely on Mars.

6.1.3. **Cost Model of EDL System**

The case study incorporates a cost model into the design process. Cost is an important aspect of designing systems. The tradeoff between cost and performance is crucial when examining the MARS EDL system. Cost is included to ensure that a robust system is developed without simply selecting the most expensive components due to their higher performance. The cost model is composed of the cost of EDL components (structure, rockets, and sensors), fuel amount, and parachute diameter. Cost-estimating relationships for theoretical first unit space missions are used to develop the model. Table 11 shows the cost of the EDL structure, rocket technology, and control system which includes the required sensors.

Table 11: Cost of EDL components including structure, sensors, and rockets [49].

Component	Component Cost
EDL Structure	$75,000,000
Control System (includes sensors)	$3,000,000
Rockets (8)	$4,000,000
Total	$82,000,000

In addition to the component cost the rocket fuel amount and parachute diameter contribute to the system cost. Equations (36) and (37) show the determination of cost for the parachute diameter and rocket fuel mass cost.

$$Parachute\ Cost = \left(\frac{Diameter}{20}\right)\frac{\$1{,}000{,}000}{m} \tag{36}$$

$$Fuel\ Cost = Fuel\ mass\,\frac{\$1500}{kg} \tag{37}$$

During the design process the parachute diameter and fuel mass are the design parameters. The cost of the system will only be changed by the parachute cost and fuel cost during the optimization of the system. The other component costs (control system and rockets) are included in case one is determined to be underperforming and require and upgrade.

6.2. Apply Utility-Based Analysis Approach to Case Study

The steps for the utility-based analysis presented in Section 5 are applied to the Mars rover lander case study.

<u>Step 1:</u> The system and subsystems are modeled using black box models, EMS function structures [25], and a hierarchical functional model. Figure 11 is the hierarchical model used to describe the EDL System. The main function of the Entry, Descent, and Landing module is to successfully land a rover on the surface of Mars. Each level below must be accomplished in order to satisfy the function above. The four functions that will be examined in greater detail include: slow descent, control descent, and lower rover. In order to examine these subsystems in greater detail both black box models and EMS Function Structures.

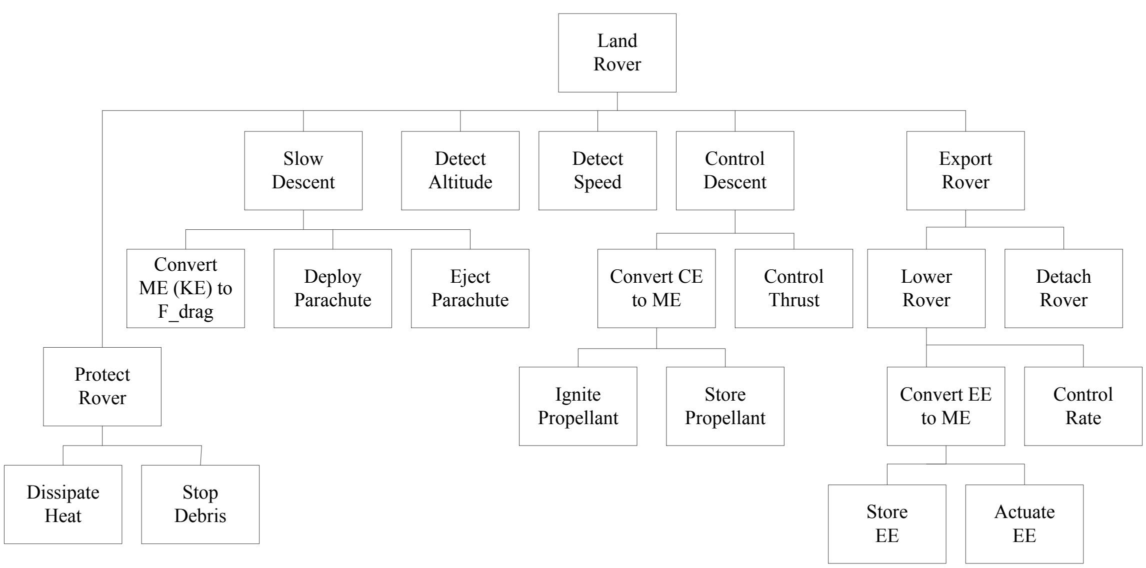

Figure 11: Hierarchical functional model of EDL System.

In order to land the rover on Mars the descent must be slowed considerably. Figure 12 shows the black box model for the parachute descent stage of the system.

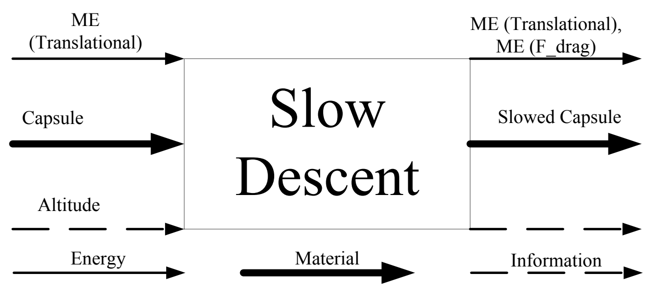

Figure 12: Black box model of parachute descent stage of EDL system.

The energy entering the system is the capsule's translational motion, while the energy exiting the system boundary is both translational and drag. The relevant signals or information during this stage is the capsule's altitude. It is crucial that the altitude sensors error be modeled for this stage. The stage is examined further with an EMS Function Structure and is shown in Figure 13.

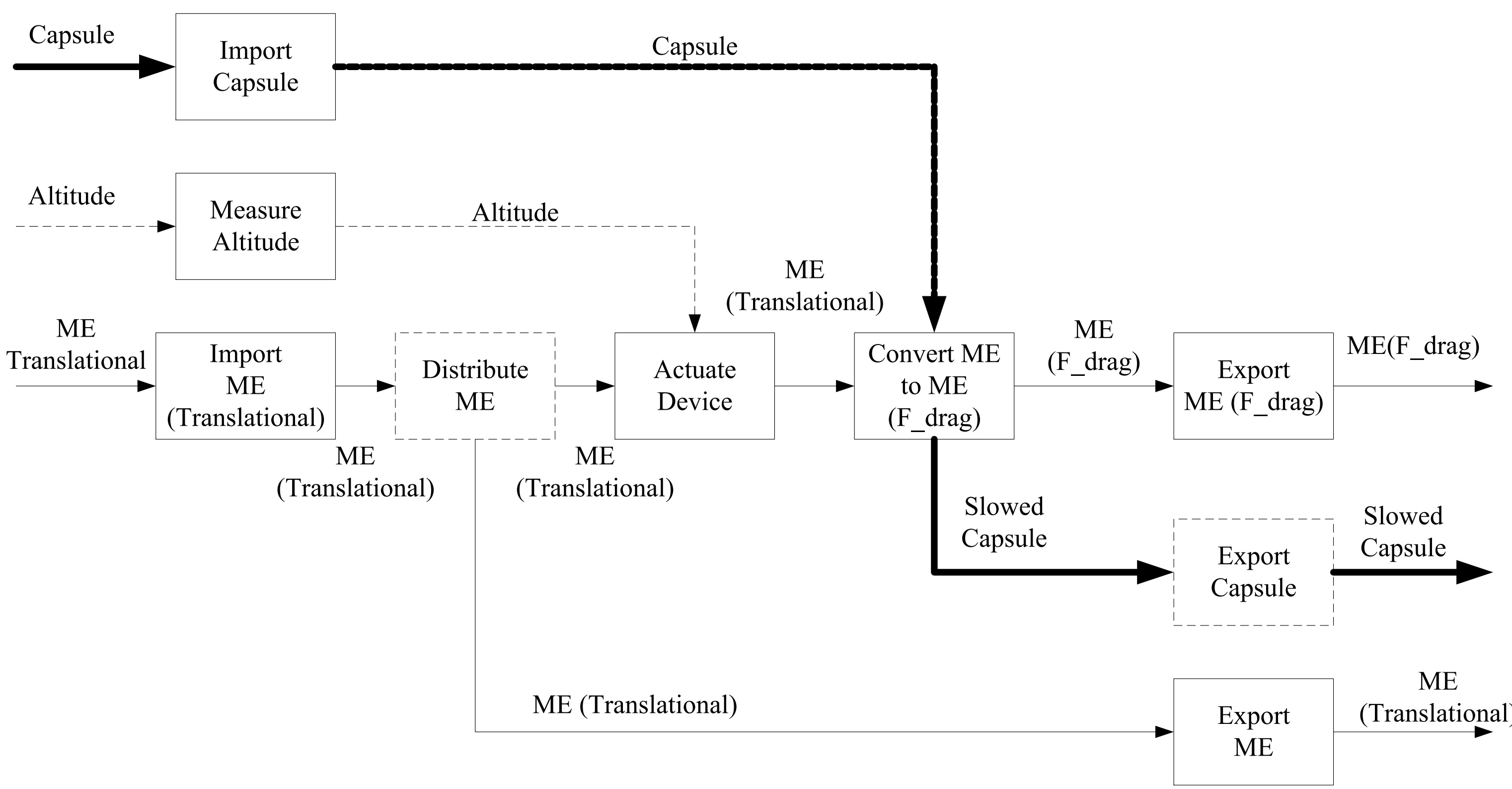

Figure 13: EMS Function Structure for parachute descent stage.

The next stage of the EDL system is the powered descent stage. Figure 14 shows the black box model of the function control descent or powered descent stage.

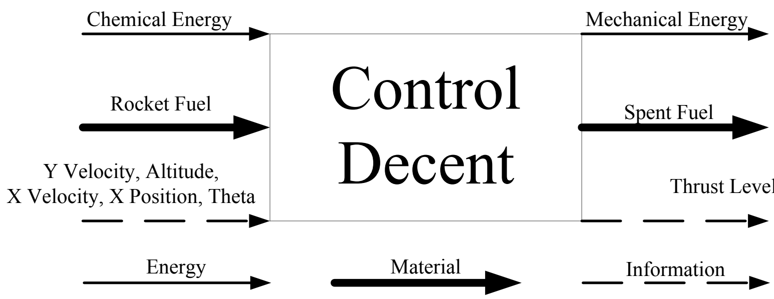

Figure 14: Black box model of the powered descent stage of the EDL system.

Much more information is required during this stage of the descent. This is due to the both the Y velocity and X position controller. It is important to include the error present in the sensors during the design of the system. The powered descent EMS Function Structure is shown in Figure 15. The EMS Function structure iterates the importance of including all of the relevant sensor noise. The function structure also shows that the rocket thrust level is a crucial component of the stage. The required thrust and actual thrust may not be exactly the same and should be modeled within the design study.

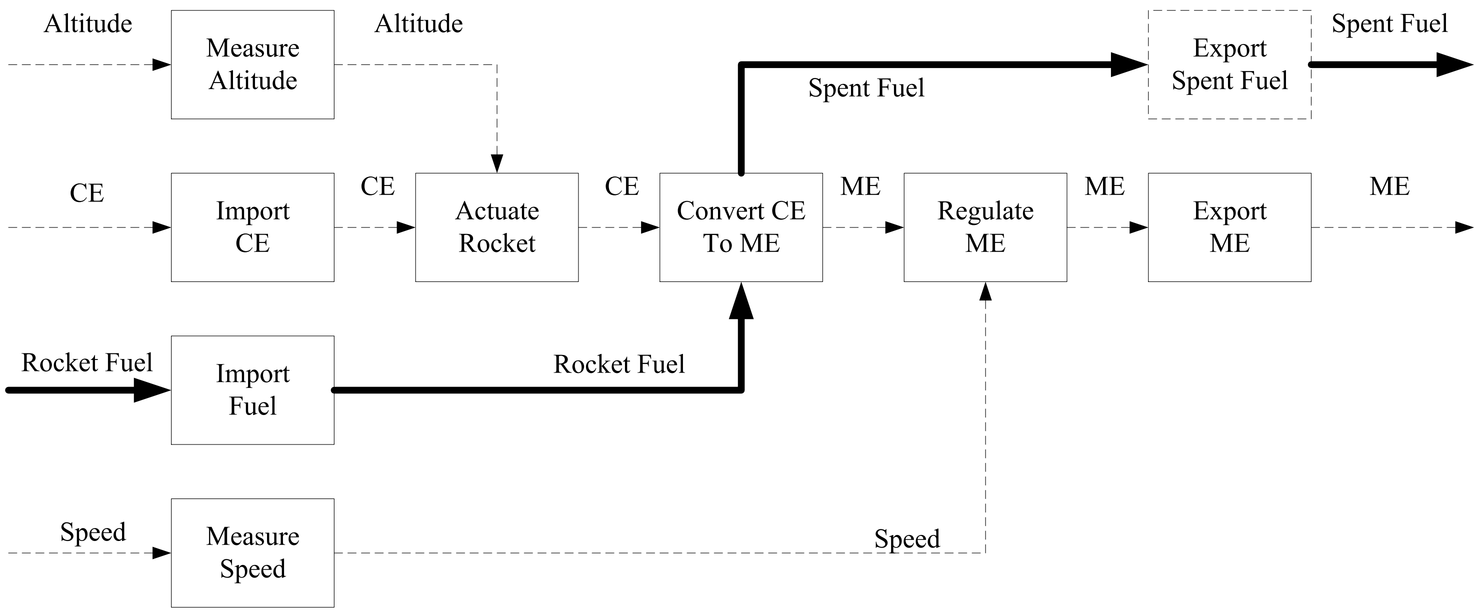

Figure 15: EMS Function Structure for powered descent stage of EDL.

The final stage to be modeled is the sky crane or lower rover subsystem. Figure 16 represents the black box model of the Sky Crane stage.

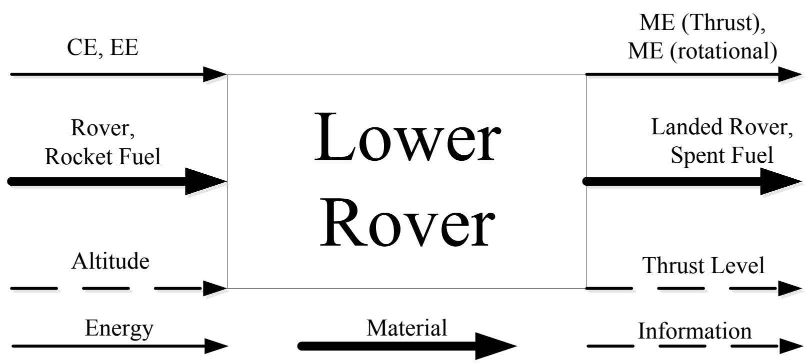

Figure 16: Black box model of the Sky Crane stage of the EDL system.

The sky crane is possibly the most important stage of the descent. The altitude must be maintained in order to ensure that the rover lands at a reasonable speed and that the EDL system does not crash into the rover as it flies away. In order to ensure this the altitude sensor error and rocket thrust uncertainty must be included within the stage. Figure 17 represents the EMS Function Structure for the lower rover subsystem. The altitude sensor is crucial for knowing the amount of thrust required and the current position of the of the rover as it is lowered. It is also crucial that the EDL maintain the altitude so that the rover lands at a reasonable speed.

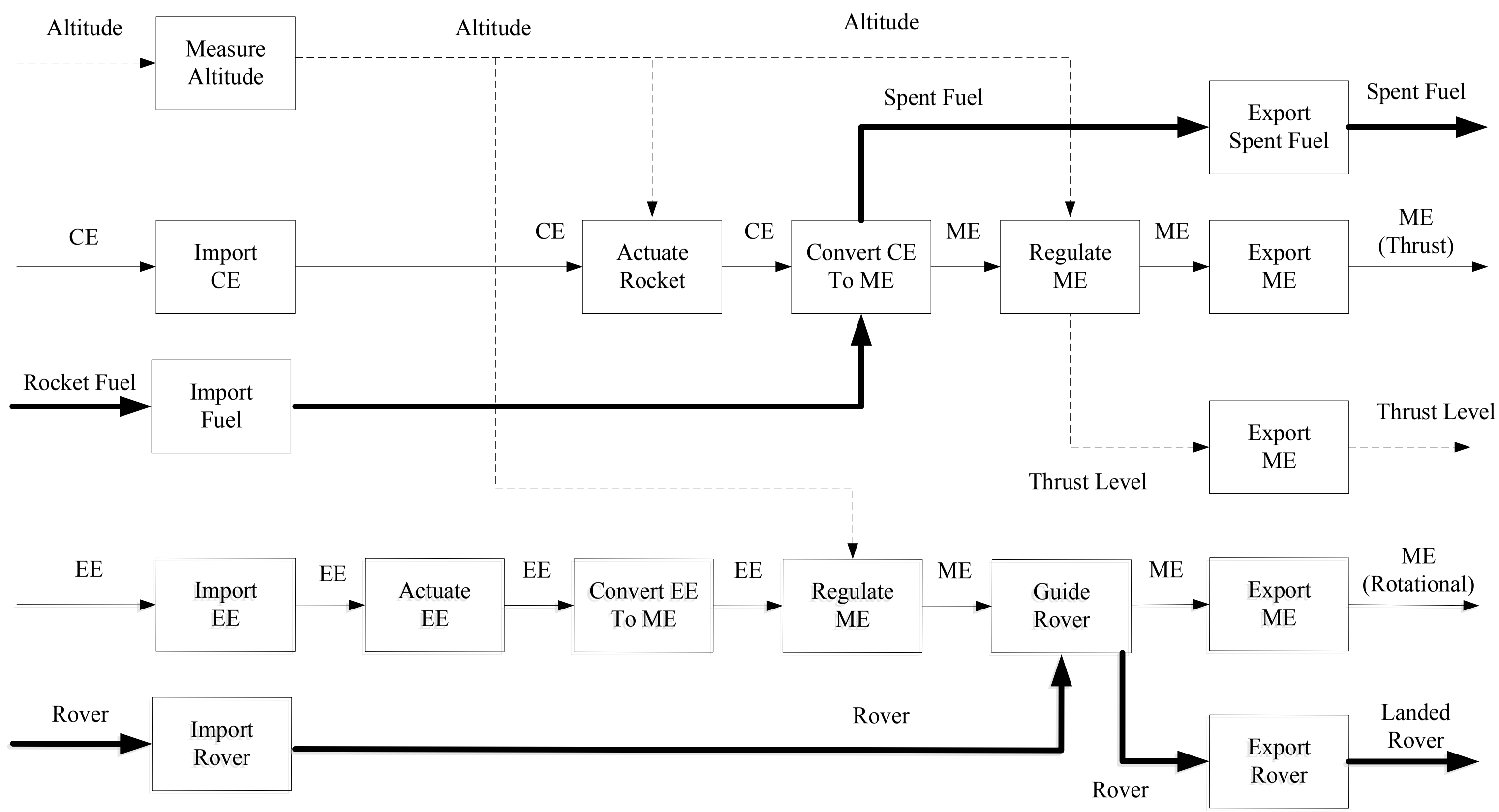

Figure 17: EMS Function Structure for Sky Crane stage of EDL System.

The developed functional models provide insight into the important internal and external perturbations. For example, from Figure 17 we observe that internal perturbations from speed and altitude sensors play an important role in the lower rover stage of the EDL system.

Step 2: The system and subsystem figures of merit are shown in Table 9.

Step 3: Using the figures of merit developed in step 2 and functional models in step 1, relevant internal and external perturbations to the system are modeled. Table 12 shows the perturbations along with their probability density function to be included in the system model.

Step 4: Utilizing the black box and EMS function structure of the entire system, it is assumed that important perturbations are included and modeled in step 3; that is, no additional internal or external perturbations are modeled.

Step 5: The subsystem figures of merit are modeled using MATLAB. The entire landing sequence is modeled in MATLAB.

Table 12: Subsystem internal and external perturbations and probability density functions modeling them.

Function	Perturbation	Probability Density Function Type	Mean	Standard Deviation
Slow Descent	Air Density Value	Normal	1	0.025
Control Descent	Rocket Output Value	Normal	1	0.075
	Y Velocity Sensor Noise (Amplitude)	Normal	1% error	0.1
	Y Velocity Sensor Noise (Frequency)	Normal	1000	50
	X Position Sensor Noise (Amplitude)	Normal	1% error	0.1
	X Position Sensor Noise (Frequency)	Normal	1000	50
	X Velocity Sensor Noise (Amplitude)	Normal	1% error	0.1
	X Velocity Sensor Noise (Frequency)	Normal	1000	50
	Theta Position Sensor Noise (Amplitude)	Normal	1% error	0.1
	Theta Position Sensor Noise (Frequency)	Normal	1000	50
	Initial Position X	Normal	100	30
	Initial Velocity X	Normal	-5	2
	Wind Velocity	Log Normal	2.191	.4724
Lower Rover	Altitude Sensor Noise (Amplitude)	Normal	1% error	0.1
	Altitude Sensor Noise (Frequency)	Normal	1000	50

Step 6: The system level utility function is developed. The two figures of merit used to measure the system utility are landing safely and system cost. One key assumptions is made when developing the system level utility function: mutual utility independence and multilinear utility function [25]. Equations (38) - (40) shows the utility function developed for landing safely, system cost, and total system utility.

$$U_1(landing\ safely) = 1 \qquad (38)$$
$$U_1(crash) = 0$$

$$U_2(cost) = -7.018 * 10^{-7} * cost + 58.80 \qquad (39)$$

$$U_{Total}(U_1, U_2) = .75U_1 + .15U_2 + .1U_1U_2 \qquad (40)$$

Step 7: The optimal system design is computed by using optimization techniques to find the maximum system utility. Latin Hypercube sampling is used to sample the perturbations developed in step 3. Table 13 shows the results obtained using optimization.

Table 13: Results from optimization of total system utility.

E(Utility)	0.620
Diameter	15.28 m
Fuel	459.9 kg

The calculated landing percentage is 65.8% with a 95% confidence internal from 62.7% - 68.7%. The current system performance is unacceptable and needs to be examined more.

Step 8: A one-at-a-time sensitivity analysis is performed on the uncertain parameters. That is, one parameter is varied while the others are maintained at their nominal condition. Table 14 shows the parameters examined in the analysis.

Table 14: Parameters examined in the sensitivity study.

Parameter	Nominal Value
Altitude Error (Amp.,Freq.)	1%, 1000 rad/s
Y Velocity Error (Amp., Freq.)	1%, 1000 rad/s
X Position Error (Amp.,Freq.)	1%, 1000 rad/s
X Velocity Error (Amp., Freq.)	1%, 1000 rad/s
Theta Error (Amp., Freq.)	1%, 1000 rad/s
Wind Speed	10 m/s
X Position	100 m
X Velocity	-5 m/s

Figure 18 shows the sensitivity analysis of landing safely for two perturbations. The output thrust and air density perturbations are examined due to their small range of operation. The other thirteen perturbations can successfully land for a much larger range of perturbations and are not included on the plot.

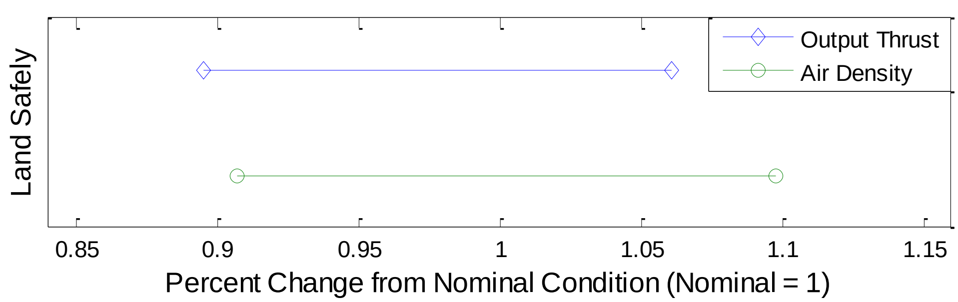

Figure 18: Range of landing safely versus change from nominal of parameters.

The speed of the EDL system at parachute ejection or slow descent subsystem is examined in Figure 19. Only the air density parameter is examined because the other parameters do not interact with the system until after the parachute has been ejected.

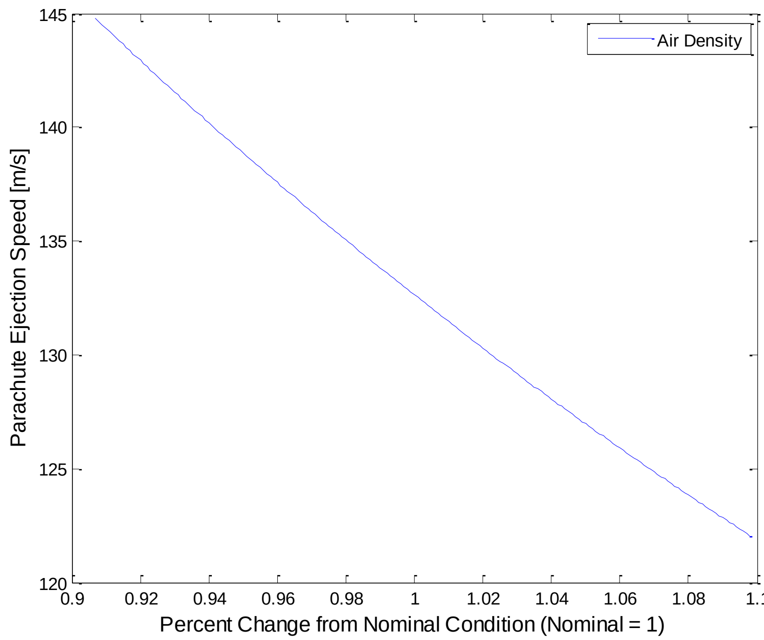

Figure 19: Parachute ejection speed (stage: parachute stage) under different parameter conditions.

Next analysis plots examine the control descent subsystem of the EDL system. Two characteristics are important when examining the powered descent stage: X landing position and Y overshoot percentage. Figure 20 - 22 demonstrate the variation in X landing position under various parameter conditions. Altitude error and Y velocity error are not examined due to their negligible effect on the x landing position.

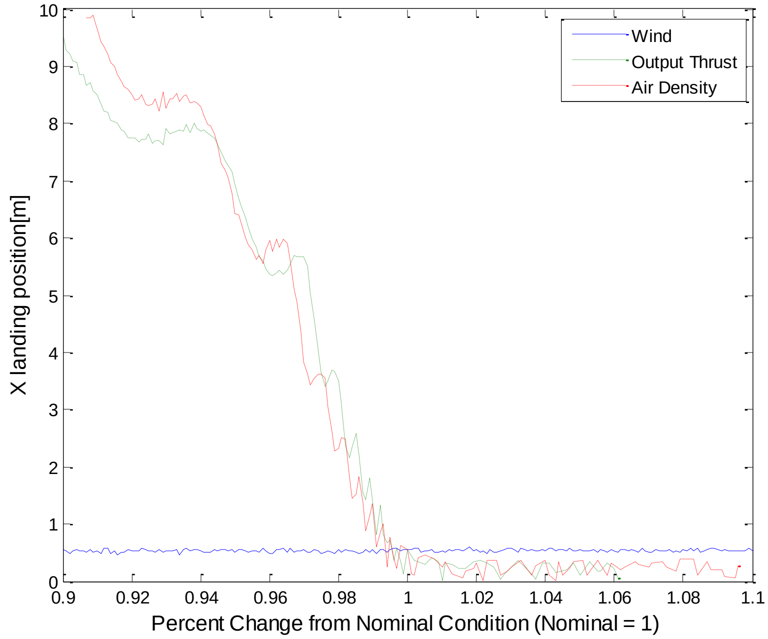

Figure 20: X landing position (stage: powered descent) under different parameter conditions. Analysis of wind, output thrust and air density.

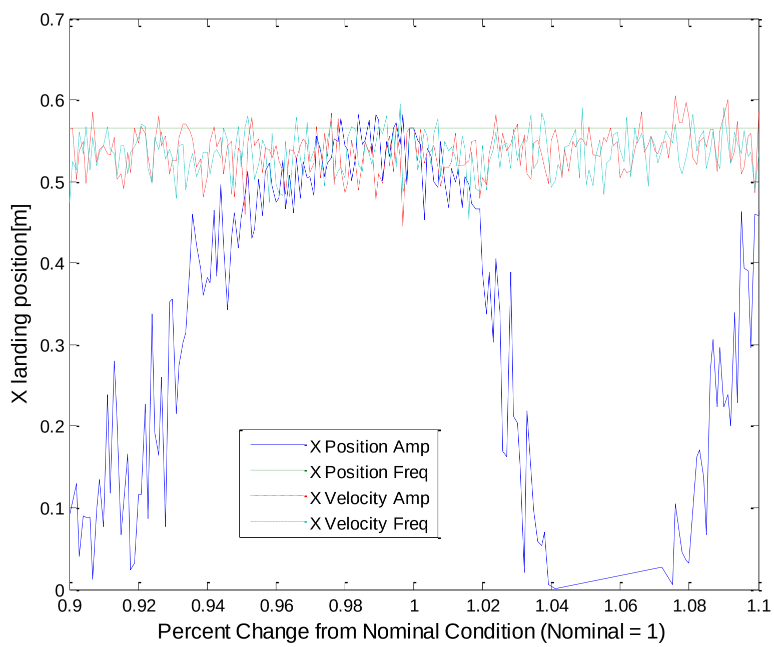

Figure 21: X landing position (stage: powered descent) under different parameter conditions. Analysis of x position sensor and x velocity sensor.

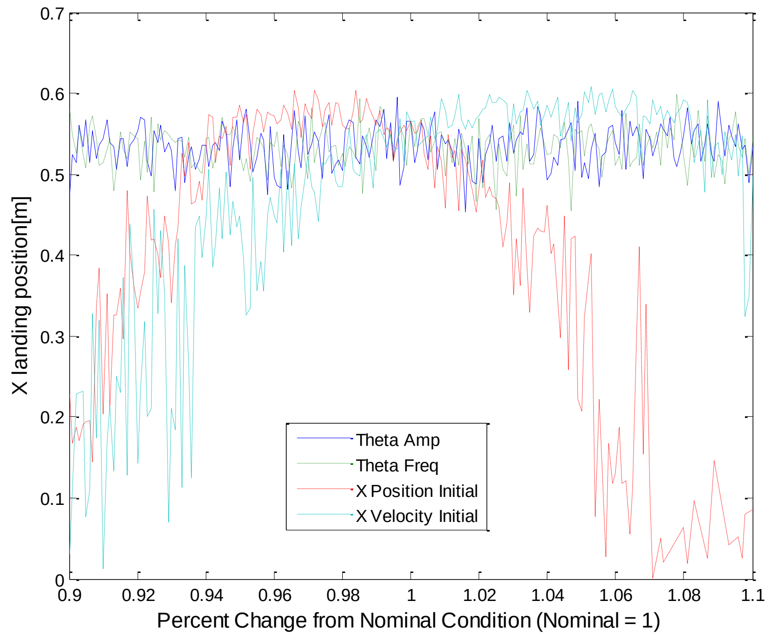

Figure 22: X landing position (stage: powered descent) under different parameter conditions. Analysis of theta sensor and x initial conditions.

Figure 23 and 24 show which parameters change the Y overshoot. X position error, X velocity error, Theta error, X initial position, and X initial velocity are not included due to the minimal change in Y overshoot when these parameters are changed.

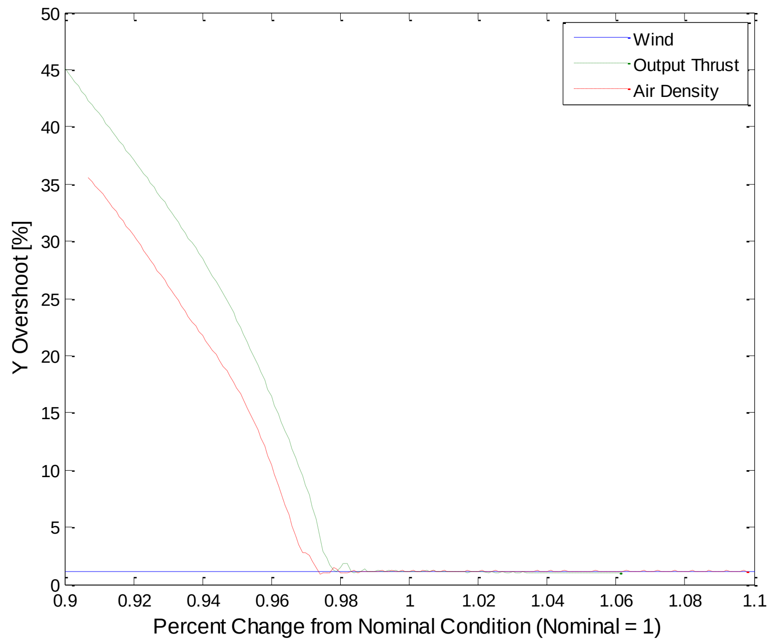

Figure 23: Y overshoot percentage (stage: powered descent) under different parameter conditions. Analysis of wind, output thrust, and air density.

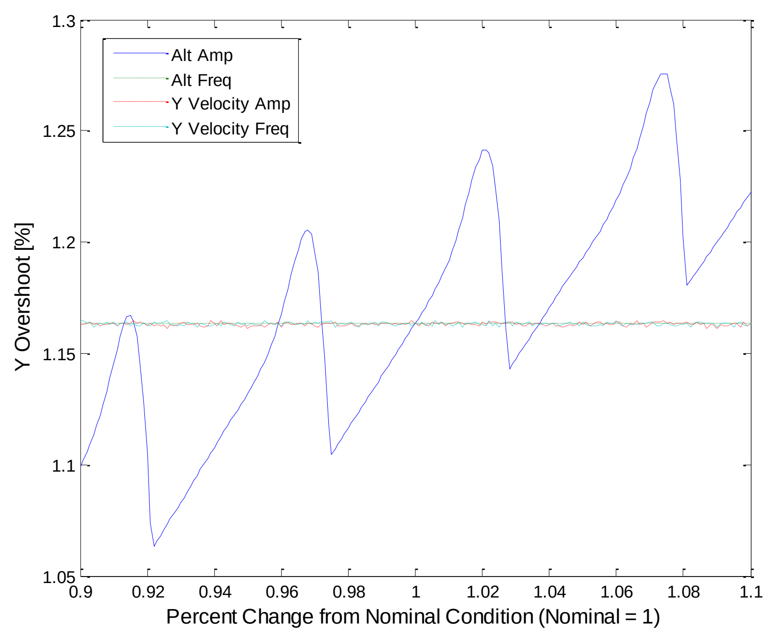

Figure 24: Y overshoot percentage (stage: powered descent) under different parameter conditions. Analysis of altitude sensor and y velocity sensor.

Analysis plots are generated for the landing speed to examine the sky crane subsystem. Figure 25 and 26 show the landings speed variation versus change from nominal condition. X position error, X velocity error, Theta error, X initial position, and X initial velocity are not included because the landing velocity remains within 0.01 m/s under their change from nominal.

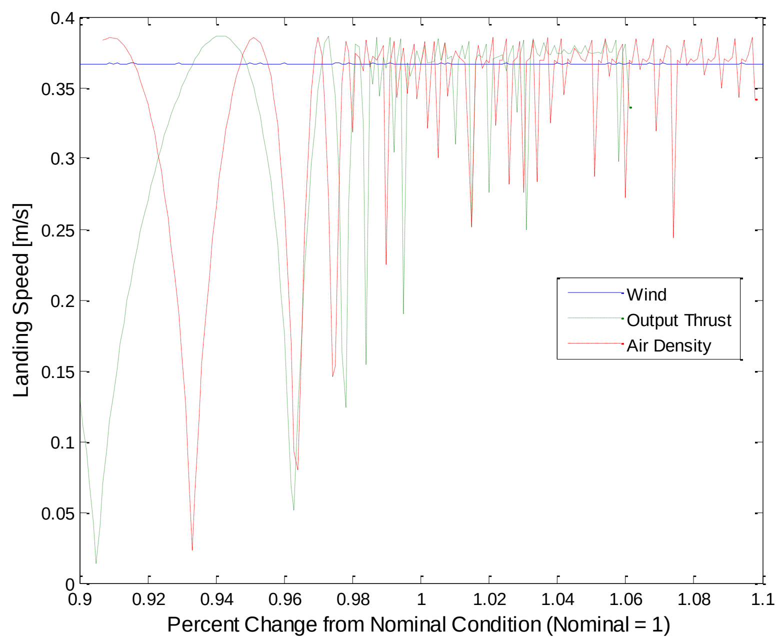

Figure 25: Landing speed (stage: sky crane) under different parameter conditions. Analysis of wind, output thrust, and air density.

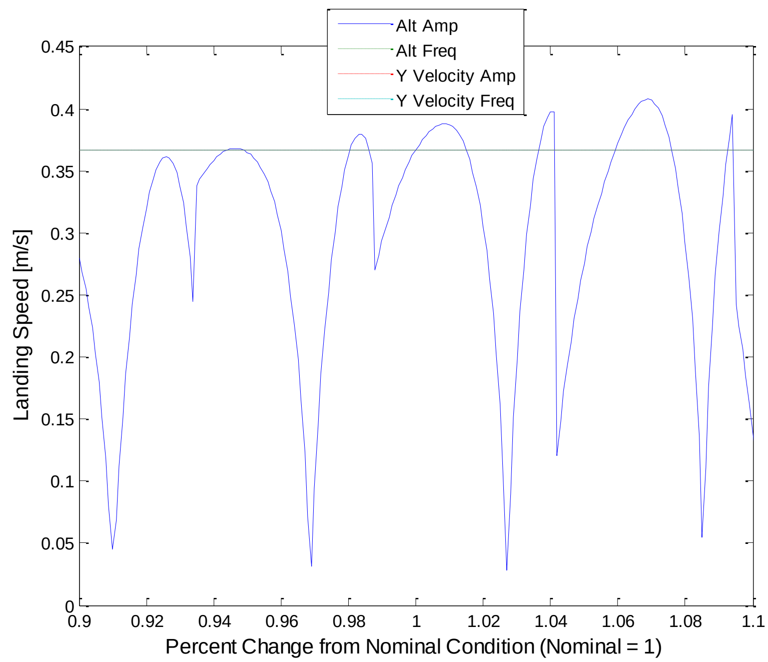

Figure 26: Landing speed (stage: sky crane) under different parameter conditions. Analysis of altitude sensor and y velocity sensor.

Step 9: The one at a time sensitivity analysis generated in step 8 provide crucial information into which system and subsystem functions are susceptible to changes in perturbations. Figure 18 shows landing the rover successfully on the Martian surface is most susceptible to changes in air density and output thrust. The range of successful landing for changes in air density are 0.91 to 1.09 of nominal value. Figure 27 shows the probability density function of the air density.

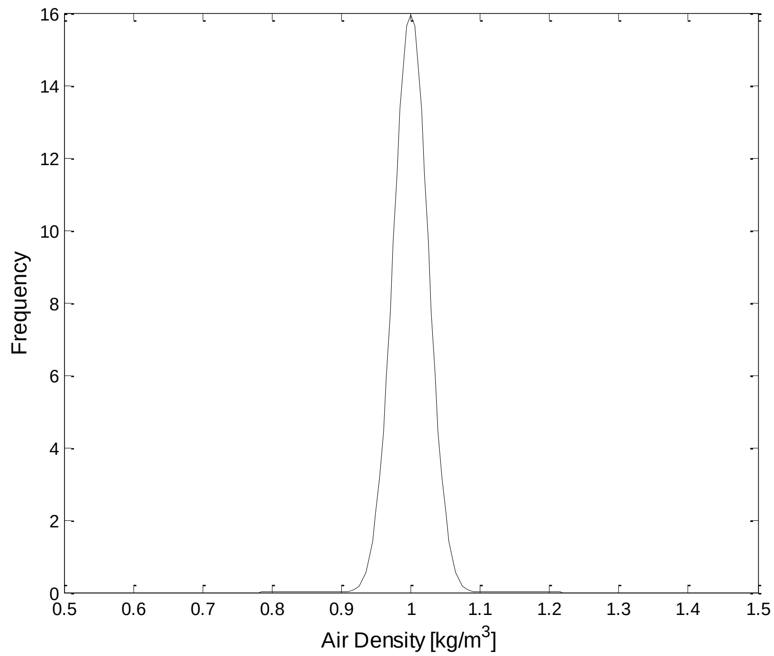

Figure 27: Probability Density Function of the air density.

The probability that the air density is outside acceptable range is nearly zero. This shows that the designed system is capable of maintaining function within the anticipated perturbation. The cost that would be required to decrease the error present in the air density model is far too high to justify it.

The rocket output uncertainty is of much higher concern. The successful range for landing the rover on Mars for the rocket output ranges from 0.895 to 1.06 of the nominal value. The probability of the rocket output to be outside the land safety range is too high. Figure 28 represents the probability density function of the rocket output. The system is unable to handle the current probability density function for rocket output. Investigation into possible solutions will be examined in the next step.

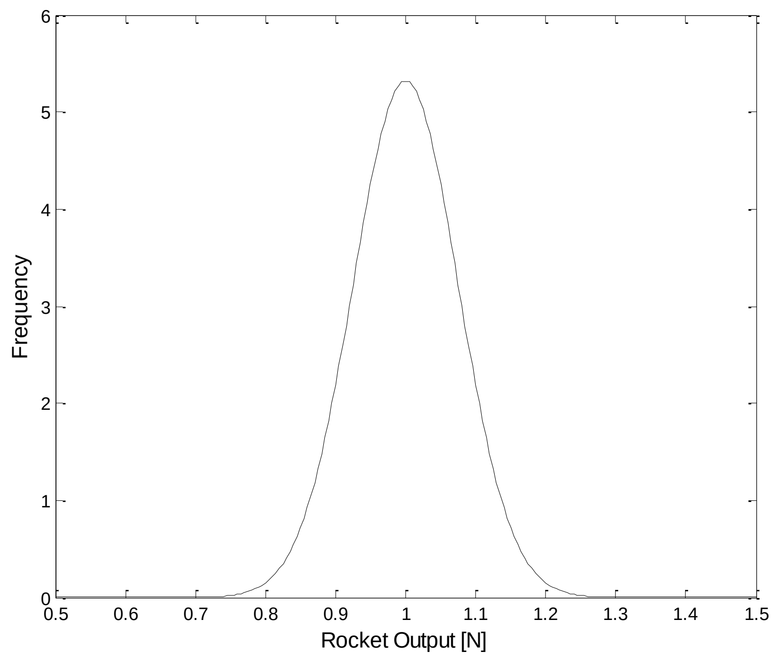

Figure 28: Probability Density Function of the rocket output.

The Powered Descent stage is concerned with both X landing position and Y overshoot percentage. Figure 20 shows that the X landing position is again susceptible to parameter changes in air density and output thrust while able to handle large changes in wind speed. Figure 21 and 22 exhibit that the powered descent stage is able to handle parameter changes in to all of the sensor error amplitudes and frequencies. The Powered Descent stage is also capable of handling changes within the X initial position and X initial velocity.

The Y overshoot percentage is highly susceptible to parameter changes in the air density and output thrust similar to the X landing position. The Powered Descent Stage is capable of handling the anticipated values of air density but the output thrust is again of higher concern. Figure 24 shows that the Y overshoot is susceptible to parameter changes in the altitude amplitude sensor noise. The max Y overshoot value is approximately 1.27%. This still leaves plenty of room for overshoot and is within acceptable range.

The sensor noise becomes more of an issue when examining the Sky Crane subsystem. Figure 26 shows how the altitude amplitude sensor noise is of greater concern than originally thought. As the amplitude is increase the landing speed increases. The landing speed approaches 0.4 m/s. While this value is still quite small, investigation into possible solutions for the altitude amplitude are investigated in the next step.

<u>Step 10:</u> Uncertainty in rocket output is shown to be of concern in step 9. This information shows where the designer(s) of the EDL should spend additional time to ensure a robust system. Adding additional rockets or redundancy to the system does not change the uncertainty of the rocket output. This would not be a suitable solution. Modularity also does not offer suitable solution to the issue of rocket output. Another option is to examine different rocket technology in order to reduce output uncertainty. For this case study, it is assumed that more expensive rocket engines can be used with a normal distribution mean of 1 and variance of 0.02. The more expensive rocket engines produce a PDF shown in Figure 29.

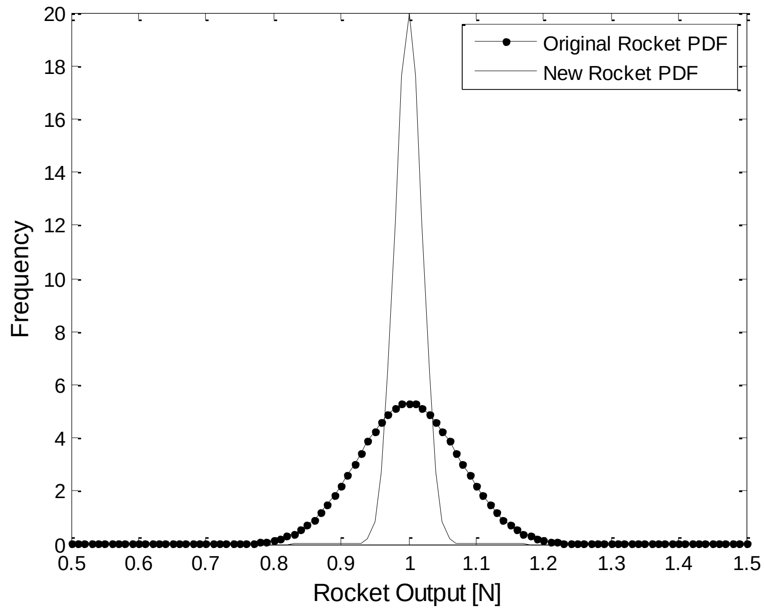

Figure 29: Probability density function of new rocket engines.

The improved characteristic of the rocket output should help the EDL system in all aspects including: landing safely, X landing position, and Y overshoot percentage. The downside of the improved rocket is the increased cost per rocket. The increased cost will be included when the optimization step is repeated.

Another issue discovered is the altitude amplitude sensor error. Step 9 shows that the Sky Crane subsystem figure of merit landing speed is highly dependent upon the altitude sensor. In order to combat this issue, a more expensive sensor is utilized to reduce the error from sensor noise. The improved sensor has an average error amplitude of 0.5%. Table 15 shows the new sensor and rocket cost for the improved EDL system. The increased cost will also be included when the optimization step is redone.

Table 15: Cost of new altitude sensor and rockets.

Component	**Cost (Per Unit)**
Original Altitude Sensor	$500,000
New Altitude Sensor Cost	$525,000
New Rockets	$530,000

Step 11: For this step, the optimization is repeated with the improved rockets and improved altitude sensor. The optimal solution with the improved sensor and rockets is shown in Table 16.

Table 16: Results from optimization using improved sensor and rockets.

E(Utility)	0.740
Diameter	15.19 m
Fuel	456.9 kg

The calculated landing percentage is 95.2% with a 95% confidence internal from 94.4% - 97.0%. The current system performance is much more acceptable then the initial design. The successful landing rate increased almost 30% over the original design. This is a huge improvement showing that the changes made to the system improved the system robustness.

The important one at a time sensitivity analysis plots are recreated using the new design parameters. The operational limits at which the EDL system successfully lands is not changed greatly from the initial design. This makes sense because the design parameters are close to their initial values. Figure 30 shows the range of successful landing for the output thrust and air density.

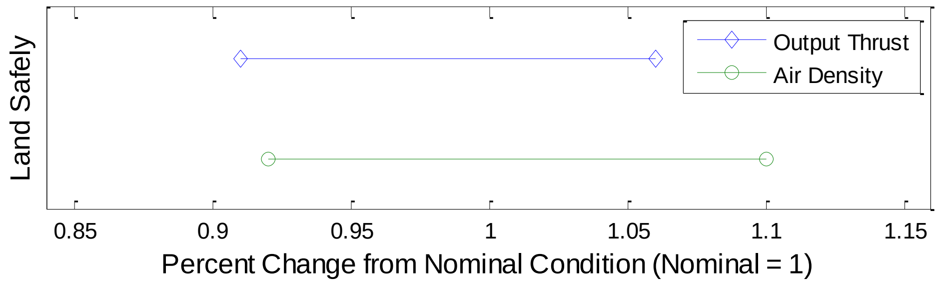

Figure 30: Range of landing safely with improved altitude sensor and rockets versus change from nominal of parameters.

The Y overshoot is reduced slightly by improving the altitude sensor and rocket noise. Figure 31 and 32 shows the Y overshoot as a function of change in nominal value of various parameters.

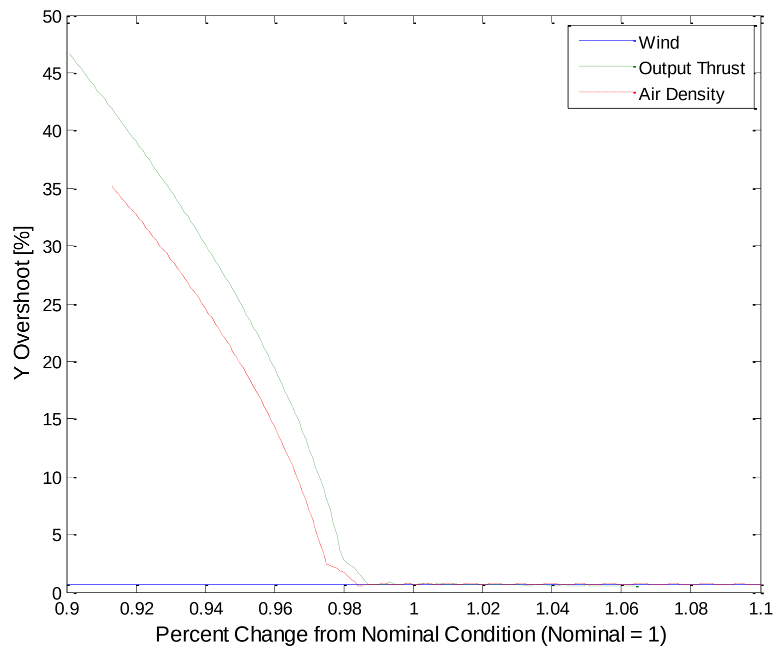

Figure 31: Y overshoot percentage (stage: powered descent) with improved sensor and rockets under different parameter conditions. Analysis of wind, output thrust and air density.

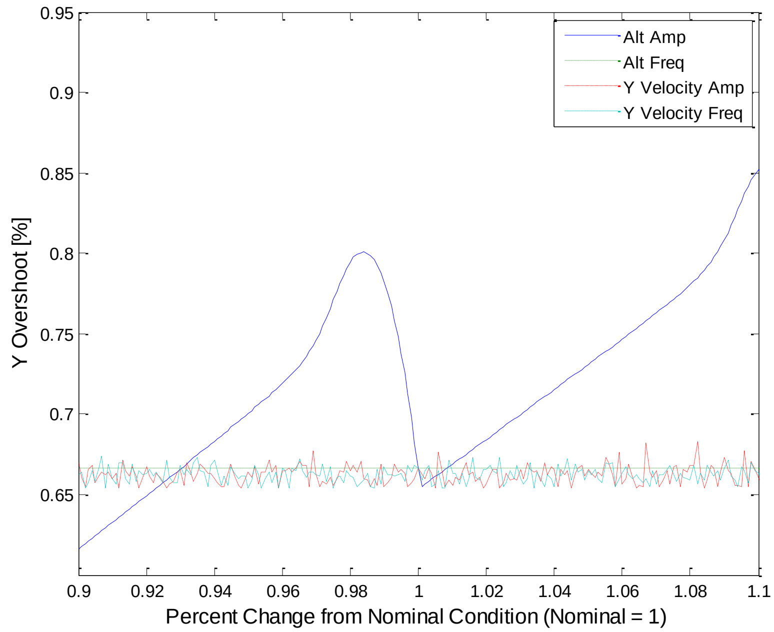

Figure 32: Y overshoot percentage (stage: powered descent) with improved sensor and rockets under different parameter conditions. Analysis of altitude sensor and y velocity sensor.

The nominal overshoot is reduced to 0.66 % while the maximum overshoot occurs when either the output thrust and air density is reduced from their nominal value. The landing speed is a critical measure of the sky crane stage of the EDL. Figure 33 shows the landing speed with the improved altitude sensor and rockets. The nominal landing speed is lowered considerably from approximately 0.36 m/s to 0.22 m/s. The reduction in landing speed ensures that the rover is not damaged when landing.

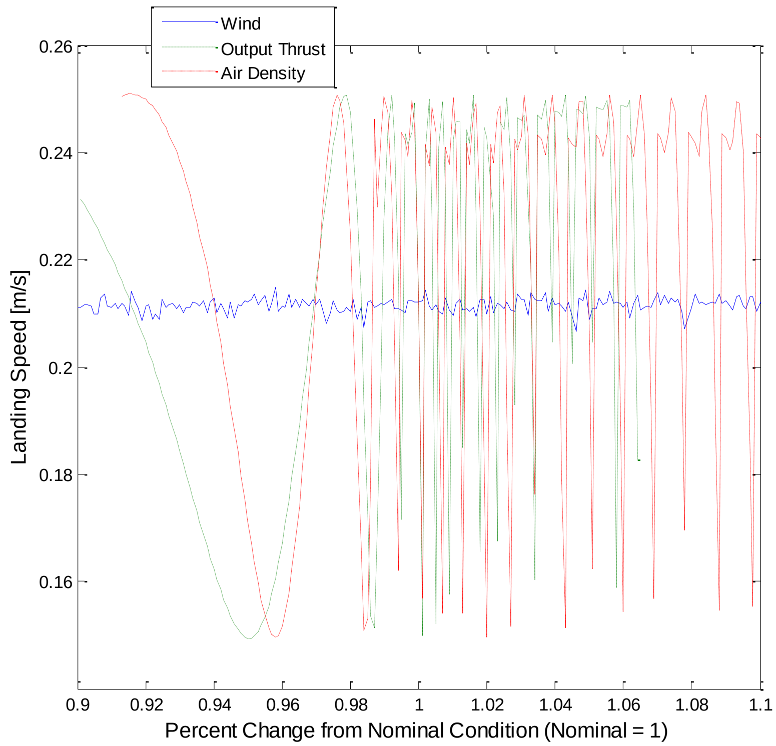

Figure 33: Landing speed (stage: sky crane) with improved sensor and rockets under different parameter conditions. Analysis of wind, output thrust, and air density.

The landing speed due to altitude sensor noise is reduced to a range of 0.15 m/s to 0.25 m/s. The previous design had a maximum landing speed of 0.4 m/s, reducing that to 0.25 m/s is a significant improvement.

The X landing position sensitivity plot with improved rockets and altitude sensor is shown in Figure 34.

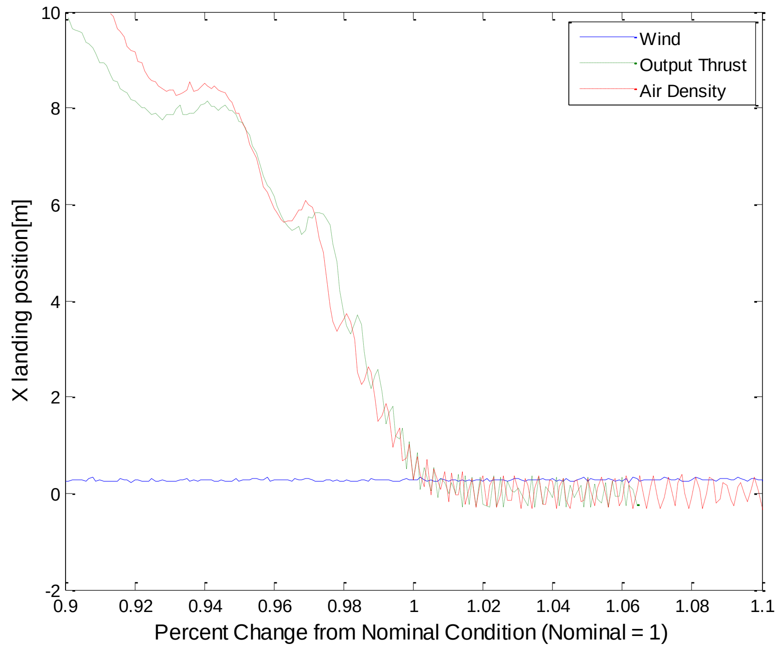

Figure 34: X landing position (stage: powered descent) under different parameter conditions with improved altitude sensor and rocket output. Analysis of wind, output thrust, and air density.

The X landing position shows why the output thrust and air density fail at approximately 90% of their nominal value. The EDL system is required to land the system within 10 m of the target site. From the x landing plot it can be seen that the EDL rarely overshoots the target. The target x position can be shifted from 0 m to -5 m in order to increase the range at which the EDL safely lands the rover. Figure 35 shows how shifting the X target to -5 m shifts the X landing more within the safe landing zone.

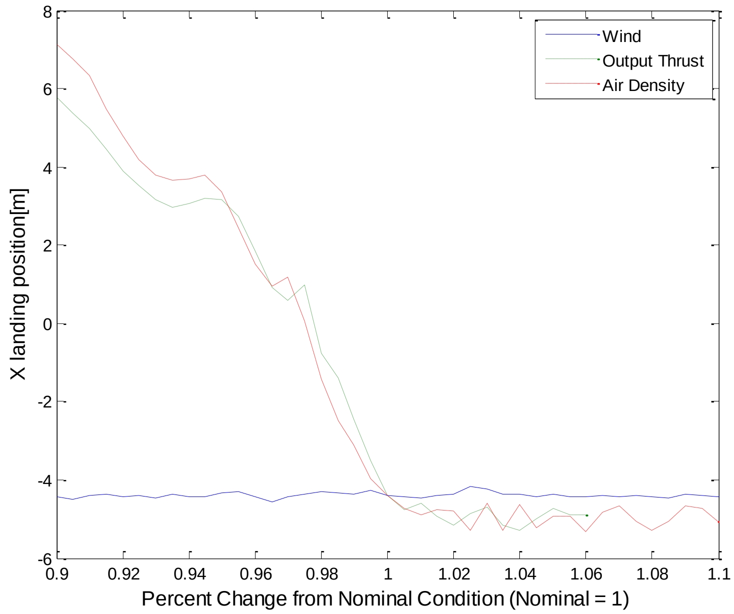

Figure 35: X Landing position with new X target landing position of -5 m.

Shifting the X target landing position increases the range at which the EDL system is able to land the rover. The optimization is rerun a third time and gives the results shown in Table 17.

Table 17: Results of optimization after adjusted x-landing position.

E(Utility)	0.742
Diameter	15.03 m
Fuel	456.9 kg

Figure 36 shows that the range of the output thrust and air density successful landings are expanded when adjusting the target X landing position.

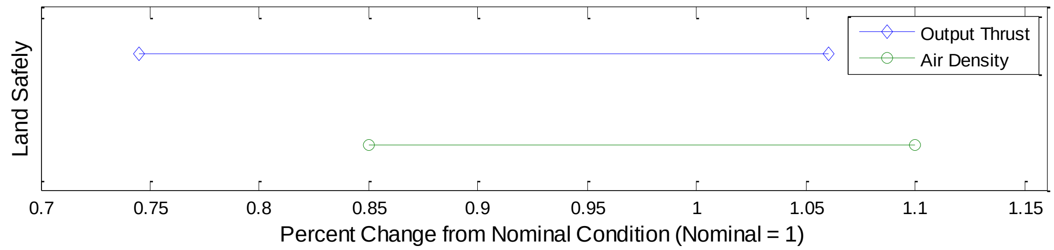

Figure 36: Range of landing safely with adjusted X target landing position.

While adjusting the X target landing position did not change the upper limits of the range of landing safely it did have a large change on the lower limit. Under these new parameters the rover lands safely 97.3% of the time with a 95% confidence interval from 96.0% to 98.2%.

6.3. Mars Rover Entry, Descent, and Landing Case Study Conclusion

The case study shows several key characteristics of the utility-based design methodology for increasing system robustness. Key results from case study:

- The EDL expected utility is increased from .620 to .742.
- The susceptible functions of the EDL were Control Descent (uncertainty in rocket output) and Lower Rover (altitude sensor error).
- The EDL successful landing percentage increased from 65.8% to 97.3%.
- The EDL system and subsystems are able to maintain function under internal and external perturbations.

Key takeaways form case study:

- Function modeling on a system/subsystem level provides insight into relevant perturbations at both the system/subsystem level.
- Sensitivity analysis of each function provides designer with insight into which system/subsystem function is susceptible to loss of function.
- The approach allows the designer to elicit a utility function with their preferences.

The creation of functional models provides the designer of the system with important information about subsystem interactions and perturbations. Both the Black Box Models and EMS Functions structures (Figure 12 - 17) show each function and their relevant perturbations. The designer then uses the knowledge gained from the function models to develop accurate PDFs of relevant perturbations and generate system models that include the relevant perturbations.

Utility Theory allows the designer to elicit a system level utility function in accordance to their preferences. This is crucial because if the designer is concerned about robustness it will show up in their preferences. Using optimization techniques with the generated PDFs and system models the design with highest expected utility is found. If the utility-based analysis method did not include any further steps the designer would not have insight into where additional improvements could be made. This is a crucial difference between MEDA and the utility-based analysis method proposed here.

The sensitivity analysis examines how a change in the perturbations values affects the system/subsystem figures of merit. This provides the designer with insight into where to spend additional time and money. Simply examining the system level figure of merit in Figure 18 would not have led to examination of the altitude sensor but solely the rocket output. This is critical because the adjustment of both the altitude sensor and rockets improved the system's ability to perform its function drastically. Examining the subsystem function in Figure 26 shows that the landing speed varied due to the error in the altitude sensor. The improved rockets and altitude sensor increased the successful landing percentage from 65.8% to 95.2%. This is almost a 30% improvement by changing only two components of the system. Additional examination of the X landing position in Figure 34 allowed for further improvement to the system. The EDL rarely overshot the landing target but may not have had enough time to get within the safe landing zone. Adjusting the X target landing position from 0 m to -5 m increased the safe landing percentage to 97.3%. The final design parameters are shown in Table 17.

Each step of the utility-based approach is an important step required to develop a more robust system. Determining the relevant perturbations and examining design changes to reduce their effect on the system generates a system that is able to maintain its function. The EDL system created in the case study exhibits robust characteristics, that is, it is able to maintain function in the presence of anticipated internal and external perturbations.

7. SUMMARY

Systems engineering continues to design and engineer more complex systems. These complex systems need to meet difficult performance characteristics while maintaining high system robustness. A general definition or method for measuring robustness is not agreed upon within the engineering community. The definition of robustness is different among scientific communities and even within specific engineering communities, while most definitions contain attributes of strength and the ability to carry out a function. For this thesis, robustness is defined as a property that allows a system to maintain its functions against anticipated internal and external perturbations [13, 21, 27].

Robust Design is a method used within engineering for developing systems with low sensitivity to noise or uncontrollable factors. Robust Design accomplishes this by utilizing the quadratic quality loss function. The quadratic loss function can be written in terms of mean and variance where the objective is to minimizing distant to target while also minimizing variance. This mean-variance approach to design forces preferences upon the designer that may not match the designers preferences. Examining Robust Design in terms of utility theory shows that the quadratic loss function has an increasing risk aversion attitude. When designing a system this risk attitude may not be rational. In order to combat this issue a utility-based analysis for increased system robustness is proposed.

The utility-based analysis for increased system robustness combines functional modeling, utility theory, and sensitivity analysis. The utility-based analysis provides steps to increase the system robustness without using a quantitative measure for robustness. Utility theory allows the designer is able to express their own preferences within the design process. The functional modeling provides a structured method for modeling the functions of the total system and its subsystems. This provides the designer with insight into the important perturbations. The sensitivity analysis allows the designer to examine which system and subsystem functions are maintaining their function and which functions should be examined in greater detail. All of these steps combined helps the designer to design more robust systems.

The Mars rover lander case study shows how a complex system can be designed using the utility-based analysis for increased system robustness. The initial design had a successful landing rate of 65.8%. Using the utility-based analysis the final design has a 97.3% successful landing rate. The functional models and sensitivity analysis pointed out the susceptible functions and which perturbations were of the greatest concern. This is an improvement of over 30%. The final Mars rover lander design is able to maintain its function under internal and external perturbations. More case studies should be performed to test the utility-based analysis for increased system robustness.

Future work includes applying the Utility-Based Analysis for Increased System Robustness on more complicated system engineering examples. This includes systems that may have more than one system level function and several subsystem level functions. Examining more complicated design examples is key in order to assess if the method helps the designer create robust systems. More case studies will also indicate where additional explanation of the method is required.

The research can be taken a step further by examining how to design resilient systems. A key step that needs to be examined in order to design resilient systems is coming up with the relevant perturbations. Possible ways to approach this could be to use the same methods that are used during concept design. These approaches could include: brainstorming, 6-3-5, morph matrixes. Resilient systems are also considered to be able to adapt to unpredictable environment changes which poses additional challenge when developing a design method.

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